3. Power factor improvement. Power measurement of single-phase receivers

The purpose of the exercise is to know the methods of measuring the power of single-phase AC receivers and the method of improving the power factor of the receiver using a capacitor.

3.1. General information

3.1.1. Instantaneous power, active power, passive, apparent
3.1.2. Power factor improvement
3.1.3. Measurement of the power of single-phase receivers by technical method
3.1.4. Determination of active power using the energy meter

3.2. Laboratory tests

3.2.1 Power measurement of single phase receiver
3.2.2 Power factor improvement

3.3. Remarks and conclusions

3.1. General information

3.1.1. Instantaneous power, active, passive and apparent

Instantaneous power, we call ratio of transient voltages and currents

$$p = u(t)i(t) \tag{3.1}$$

If the voltage u(t) and current i(t) are sinewave functions of the time:

$$u(t) = U_m \sin(\omega t + \varphi_u) \tag{3.2}$$

$$i(t) = I_m \sin(\omega t + \varphi_i) \tag{3.3}$$

where φ_u and φ_i -the initial phases of the voltage and current, after considering the equations (3.2) and (3.3), equation is obtained:

$$\begin{aligned} p &= U_m \sin(\omega t + \phi_u) I_m \sin(\omega t + \phi_i) = \\ &= U_m I_m \frac{1}{2}\left[\cos(\phi_u - \phi_i) - \cos(2\omega t + \phi_u + \phi_i)\right] \end{aligned} \tag{3.4}$$

Taking into account:

$$U_m = U\sqrt{2} \qquad \text{and} \qquad I_m = I\sqrt{2}$$

and after implementation the phase shift angle $\varphi = \varphi_u - \varphi_i$ the equation of instantaneous power takes the form:

$$p = UI\cos\phi - UI\cos(2\omega t + \phi_u + \phi_i) \tag{3.5}$$

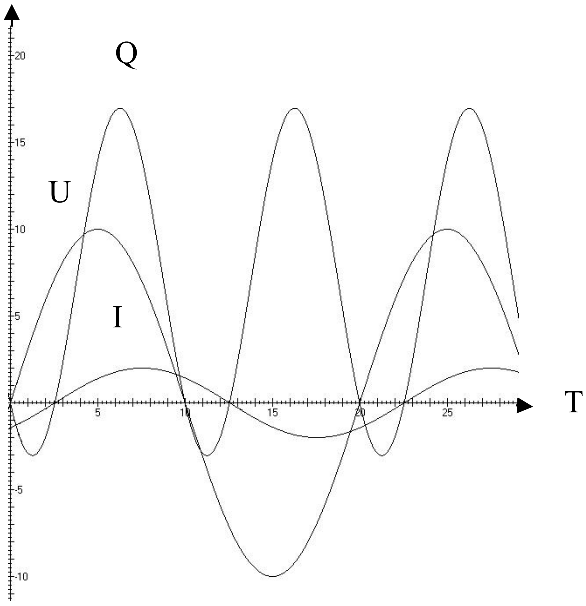

Figure 3.1. Charts of voltage, current and instantaneous power described by equations (3.2), (3.3) and (3.5)

From the formula (3.5) results that the instantaneous power p oscillates with double pulsating 2ω around the constant value of the power equal $UI\cos\varphi$. This power, equal to the average value of the instantaneous power calculated during the T period, is called the active power P:

$$P = \frac{1}{T}\int_0^T p\,dt = UI\cos\phi\,. \tag{3.6}$$

The active power unit is 1 W (Watt). The active energy corresponding to the power is energy, which is replaced in the receivers with other types of energy such as: heat, mechanical, chemical or light.

From the phasor graph shown in Fig. 3.2. it follows that

$$UI\cos\phi = UI_{cz} = P \tag{3.7}$$

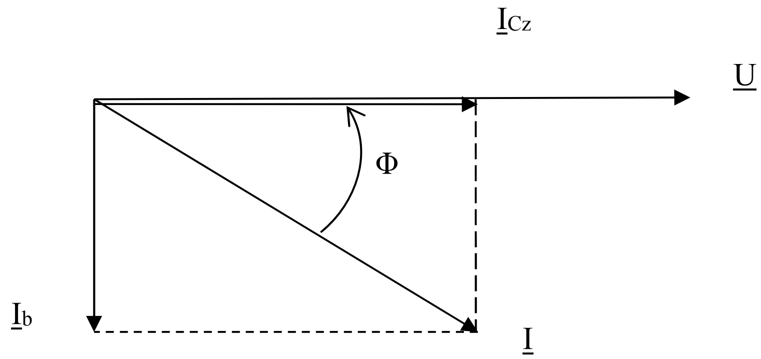

Figure 3.2. Phasor graph of voltage and current described by equations (3.2) and (3.3)

where I_{cz}^{*} - active component of current.

The second of the current components is the passive component I_b^{*} - it corresponds to the reactive power Q, which is calculated from the formula:

$$UI\sin\phi = UI_b = Q \tag{3.8}$$

The reactive power unit is 1 var (var). In contrast to active energy, passive energy is not disperse in the receiver. The corresponding its reactive power flows in the source-receiver system causing additional load on the supply line. It is needed to produce, for example, magnetic field in transformers, electric motors, etc.

The apparent power of S is called ratio values of effective voltages and currents

$$S = UI = \sqrt{P^2 + Q^2} \tag{3.9}$$

The apparent power unit is 1 VA (voltampere).

Based on relationships (3.6), (3.8) and (3.9) can be notice that the values of P, Q and S are the sides of a rectangular triangle with a acute angle of φ. It was called the power triangle (Fig.3.3).

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[*] Components: active and passive current are values in general purely mathematical cases resulting from the decomposition of the phasor I into two components. Therefore, it should not be associated with specific current values occurring in the system.

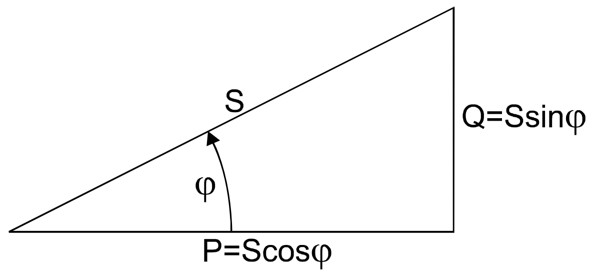

Figure 3.3. Power triangle

3.1.2. Power factor improvement

The energy balance for an AC receiver is more complicated than a DC receiver.

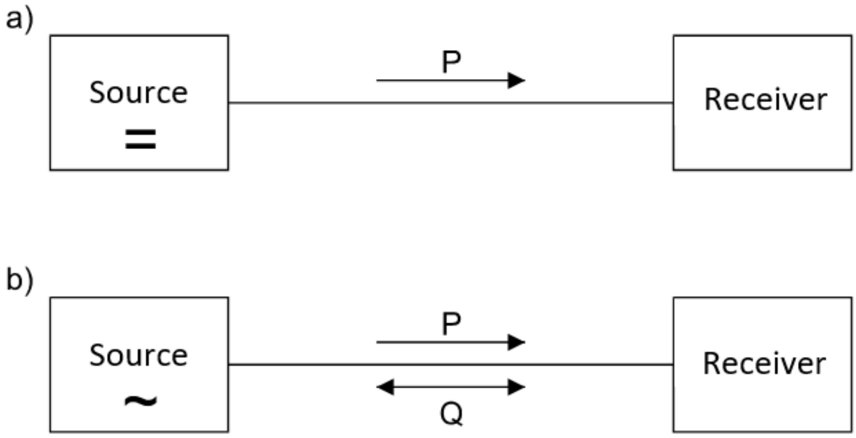

Figure 3.4. Schematic representation of the power flow in the source-receiver system:
(a) DC grid; source: battery, alternator
(b) AC grid; source: generator

For the DC receiver, the balance is only applicable to the active power P. This situation is shown in figure 3.4a. In practice, alternating current receivers are resistive-inductive and therefore (Fig.3.4b) the source (generator) must have in addition to active power, also reactive inductive power. The reciprocal relationship between the P and Q values of the receiver depends on $\cos\varphi$ of the receiver and is called the power factor, **where φ is the phase shift of the voltage and current** of the receiver, whereby (fig.3.3)

$$P = S\cos\varphi, \qquad Q = S\sin\varphi \tag{3.10}$$

The effective value of the current obtained by the receiver with the active power P can be calculated from the

$$I = \frac{P}{U\cos\varphi} \tag{3.11}$$

A lower value of cosφ, which corresponds to a greater value of Q at P = const, thus results a higher current value and the same **increase power loss ΔP and voltage drop in the transmission line Δ U** ,because formulas are applied:

$$\Delta P = I^2 R_p \qquad \Delta U \cong I Z_p, \tag{3.12}$$

where R_p, Z_p -resistance and impedance of transmission line.

A small power factor value is characterized by those receivers whose operating principle requires the generation of a magnetic field. A correspondingly large flow of reactive inductive power is reflected in this fact. To the main electrical devices, which except the active power, also received the inductive reactive power are asynchronous motors and transformers. The power factor of induction motors at rated conditions varies between 0.8... 0.97, with small loads decreasing, and at idle state reaches value below 0.3.

The improvement of the power factor (inductive reactive power compensation-decrease value of Q) involves the use of natural or special methods.

Natural ways include:

1. correct selection of engine power - the use of an asynchronous engine with too much power compared to the existing needs decreases cosφ, as the active power receiving by the motor not adapted to the mechanical load is approximately the same as for a lower power engine (with the same mechanical power requirements) – however, the higher is the reactive power,
2. avoiding the idle state of motors and transformers, because in this case cosφ has a very low value.

For special ways to improve cosφ (reactive power compensation), is the inclusion of a capacitor in parallel to the receiver, which compensates partially the inductive reactive power of the receiver.

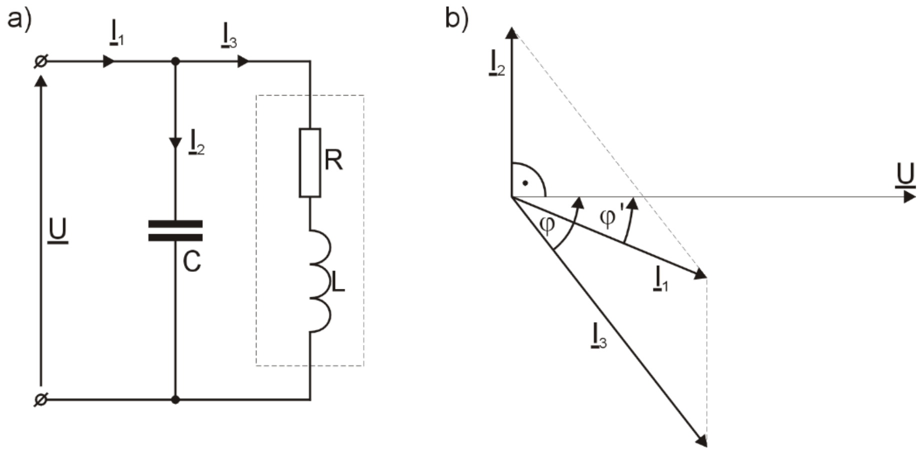

Figure. 3.5. Improvement of cosφ: a) wiring diagram; (b) phasor graph

From the phasor graph shown in fig.3.5b it is apparent that **the effective value of the current I_1 after the capacitor is switched on is less** than the effective current value before switching on the capacitor (I_3).

Note: The value of the current of the receiver I_3, thereby power receiving by the receiver remains unchanged.

At the same time, the condition is fulfilled:

$$\cos\phi' > \cos\phi \tag{3.13}$$

With the unchanged active power of single-phase receiver ($UI_1 \cos\phi' = UI_3 \cos\phi$), the capacitance of capacitor, which have to be switched on, to "improve" the power factor to value $\cos\phi'$ is:

$$C = \frac{P}{\omega U^2}(\mathrm{tg}\phi - \mathrm{tg}\phi') \tag{3.14}$$

This compensation method is called individual compensation (for a single receiver). This compensation of reactive power can also be used for group of receivers (capacitor assemblies in power distribution units), as well as centrally in the main supply stations of the energy system.

3.1.3. Measuring the power of single-phase receivers by technical method

This measurement is carried out by watt-meter, voltmeter and ammeter in the circuits as in Fig. 3.6a or 3.6b.

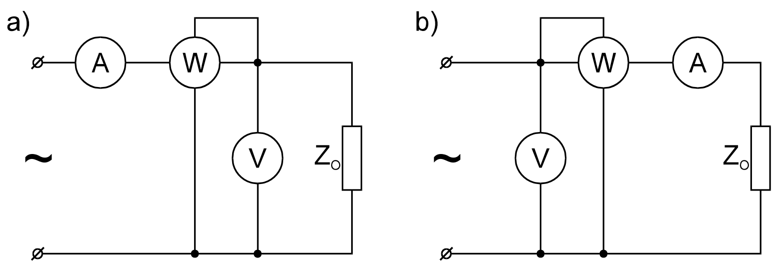

Figure 3.6. Diagrams of single phase power measurement:
(a) circuit for low-impedance receivers Z_0;
b) circuit for high-impedance receivers Z_0

In the circuit as in Fig. 3.6a the active power measured by the wattmeter is greater than the power of the receiver by the power emitted in the voltmeter and the voltage circuit of the wattmeter

$$P_w = P + \frac{U_v^2}{R_{wn}} + \frac{U_v^2}{R_v} \tag{3.15}$$

with R_{wn}, R_v - resistance, respectively of the voltage circuit of the wattmeter and voltmeter, Z_0 - impedance of the receiver.

In the circuit as in Fig. 3.6b the active power measured by the wattmeter is greater than the active power of the receiver by the power emitted in the current circuit of the wattmeter and in the amperemeter.

$$P_w = P + I_a^2 R_{wi} + I_a^2 R_a \tag{3.16}$$

with R_{wi}, R_a - resistance, respectively of the current circuit of the wattmeter and ammeter, Z_0 - impedance of the receiver.

The choice of the specified circuit must be made so as to minimize the additional measured powers, and thus the arrangement a) should be applied at low, and the arrangement B) at high impedance of the receiver.

The active power measured with the wattmeter is determined from the formula:

$$P_w = k_w \alpha \tag{3.17}$$

where k_w - constant of the wattmeter [W/plots], α - number of plots corresponding to the inclination of the meter gauge.

The constant of the wattmeter is as follows:

$$k_w = \frac{U_{zn} I_{zn} \cos\varphi_{zn}}{\alpha_{zn}} \tag{3.18}$$

where U_{Zn}, I_{Zn} - rated value of the voltage and current ranges of the wattmeter,
$\cos\varphi_{Zn}$ - rated power factor of the wattmeter (if not given, i.e. $\cos\varphi_{zn}= 1$),
α_{Zn} -rated number of scale plots of the meter.

The voltmeter and ammeter are always switched on with the power meter to avoid overloading the voltage and current circuit of wattmeter.

The apparent power of the receiver can be determined by the voltmeter and amperemeter:

$$S = U_v I_a \tag{3.19}$$

and reactive power

$$Q = \sqrt{S^2 - P_w^2} \tag{3.20}$$

3.1.4. Determination of the active power by using an inductive active energy meter

For the active energy meter the number of revolutions of its shield is proportional to the active energy A of the receiver. If we measure the time Δt of a certain number of revolutions N, the active energy can be calculated from the formula:

$$A = \frac{N}{c_L} \cdot 3{,}6 \cdot 10^6 \ [\mathrm{J}] \tag{3.21}$$

where C_L - the constant of energy meter $\left[\frac{rev}{kWh}\right]$

The active power of the receiver is determined by formula:

$$P = \frac{A}{\Delta t} = \frac{N}{c_L \Delta t} \cdot 3{,}6 \cdot 10^6 \ [\mathrm{W}] \tag{3.22}$$

3.2. Laboratory tests

3.2.1. Power measurement of single phase receiver

The active power of the single-phase receiver shall be measured by using the active energy meter and the wattmeter in the system presented in Fig. 3.7.

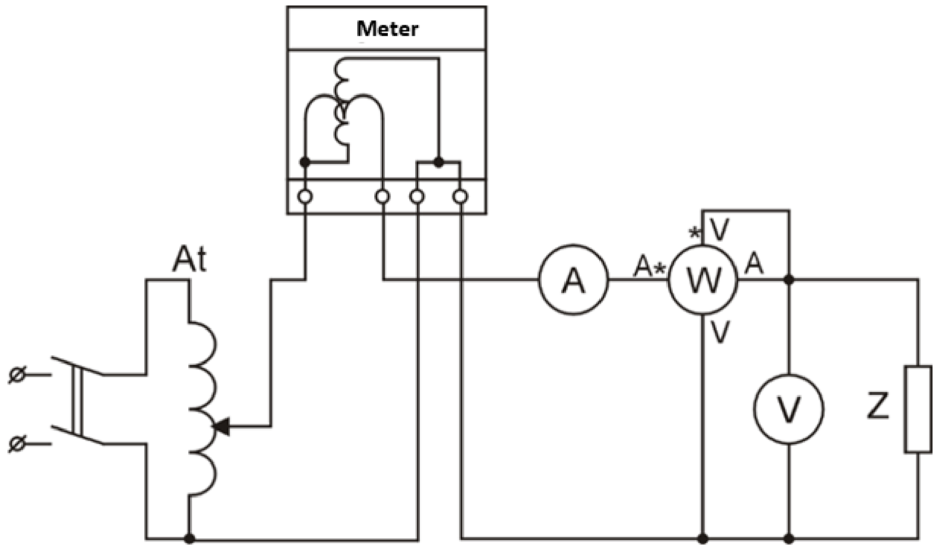

Figure 3.7. System for measuring the active power of a single phase receiver
At - autotransformer, A - ammeter, W - wattmeter, V - voltmeter

Measurements carried out for different receivers. The results of the measurements and calculations write in table 3.1.

Provide examples of calculations according to formula 3.22.

Based on the results of measurements and calculations, draw a phasor graph of currents and voltages for each type of receiver.

Table 2. 3.1.

Receiver	U_V	I_a	P_L			P_W			S	Q	cosφ	φ
			N	Δt	P_L	α	K_W	P_W				
	V	A	-	S	W	plots	W/plots	W	VA	var	-	deg
R												
R												
L												
C												
Rl												

3.2.2. Power factor improvement

Perform measurements of voltages, currents and power without capacitor and with the switch on capacitor in the system shown in Fig. 3.8.

Measurements performed with open and closed circuit breaker for different capacitor capacity values. The results of the measurements and calculations write in table 3.2. Provide examples of calculations according to the following formulas:

$$\cos\phi = \frac{P_w}{U_v I_3}, \qquad \cos\phi' = \frac{P_w}{U_v I_1}$$

$$C = \frac{I_2}{2\pi f U_V} \qquad \text{Or} \qquad C = \frac{P_w}{\omega U^2}(\mathrm{tg}\phi - \mathrm{tg}\phi')$$

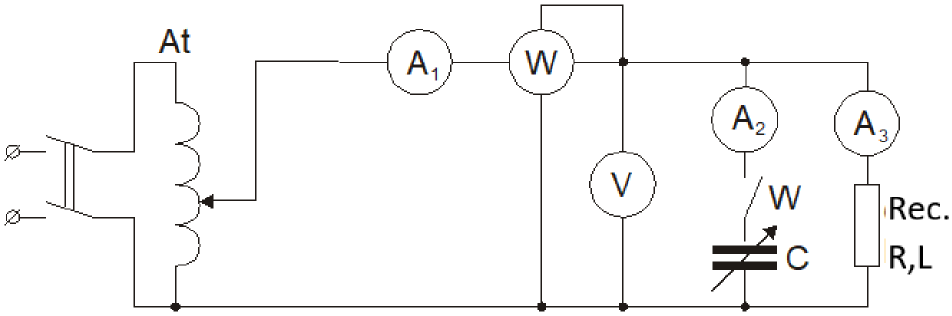

Figure 3.8. Power factor improvement system
A_1, A_2, A_3 - ammeter, V- voltmeter, W - wattmeter,
C - capacitor, Rec. R, L - receiver

Based on the results of measurements and calculations, perform an phasor graph of currents and voltages for each test system.

Note: the R, L receivers are a serial connection of R and L elements.

Table 3.2

No.	U_v	I_1	I_2	I_3	P_W			cosφ	cosφ'	φ	φ'	C
					α	K_W	P_W					
	V	A	A	A	plots	W/plots	W	-	-	deg	deg	μF
1												
2												
3												
4												
5												

3.3. Remarks and conclusions

Based on theoretical material and results of measurements and calculations:

A) for p.3.2.1 compare the power measurement results obtained by using the energy meter and the wattmeter and justify possible differences in the measurement results

b) for p.3.2.2 compare the results of theoretical considerations with the measurement results, indicate increasing of effective current value in the circuit when "overcompensating" (too high C value).

Audit questions:

1. Powers in alternating current circuits. Provide their physical interpretation and units.
2. Justify the necessity of improvement the power factor.
3. Bring out the formula (3.14).
4. Discuss how to measure power P, Q, S.

Literature

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