

## **42. Direct current. Laws, theorems, calculation methods**

The purpose of the exercise is to practice the laws and theorems for direct current circuits and test their application.

### **42.1. General information**

### **42.2. Laboratory tests**

42.2.1. Ohm's law

42.2.2. Characteristics of voltage and current sources

42.2.3. Kirchhoff's laws

42.2.4. Method of circuit's transformation

42.2.5. Superposition method

42.2.6. Thevenin and Norton theorems

### **42.3. Remarks and conclusions**

### 42.1. General information

The main issue in electrical engineering is to analyze electrical circuits by calculating currents and voltages in order to determine operation and behavior of various electrical systems.

Electrical circuits are made by combining various components (e.g. energy sources, resistors) which are indivisible part in functional terms. Each element has clamps to which the wires are connected.

The element is called linear when it can be described by the linear equation. Circuits with only linear elements are called linear circuits.

Energy processes in electrical circuits are:

- generation from other forms of energy (chemical, e.g. cells, accumulators; mechanical - generators; light - photovoltaics) - these are active elements called energy sources,
- energy dissipation - these are passive elements such as resistors, where electric current is transformed into thermal energy and is dissipated,
- energy accumulation - these are passive elements in which energy is stored in an electric field (e.g. capacitors) or magnetic field and can be fully returned.

DC circuits are supplied by voltage and current sources at constant in the time voltages source and currents source. Resistors are the basic elements of passive DC circuits. The constant currents and voltages are labelled in capital letters.

**Resistor**

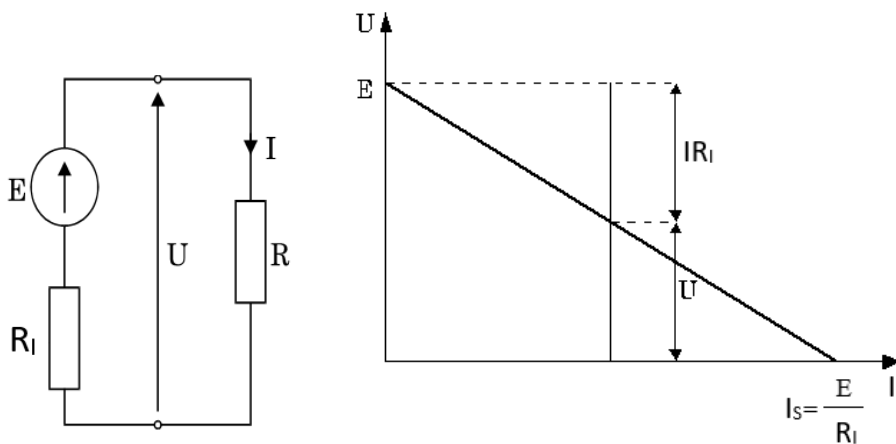
The ideal resistor is an element in which only the process of energy dissipation occurs (no generation or accumulation of energy). The parameter characterizing the ideal resistor is resistance  $R$ . Resistance of the linear resistor is constant. The resistance of a homogeneous conductor with a fixed section is directly proportional to the length of the conductor  $L$ , inversely proportional to the sectional area  $S$  and depends on the conductivity  $\gamma$  of the material, which characterizes the relevant materials in terms of electrical conductivity.

$$R = \frac{l}{\gamma S} \tag{42.1}$$

**Energy sources**

– **Voltage source**

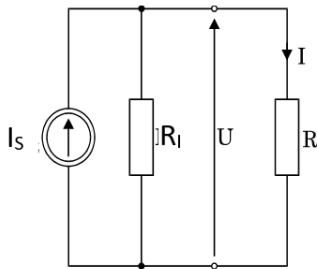
An energy source with a serial connection of an ideal source of voltage and resistance called an internal resistance known as a real voltage source. An ideal source of voltage is the energy source with an internal resistance equal to zero. The difference in potentials of the terminals of ideal source is called the source voltage  $E$ .



**Figure 42.1.** Voltage source - diagram and characteristic

$$U = E - IR_i \tag{42.2}$$

– **Current source**



The energy source with a parallel connection of the ideal current source and resistance is called the real current source. An ideal source of current is a component of an electrical circuit providing constant current and very high internal resistance. The internal resistance of the ideal current source is infinitely large.

**Figure 42.2.** Current source - diagram

$$I = I_s \frac{R_i}{R + R_i} \quad (42.3)$$

**Fundamental laws**

– **Ohm's law**

The voltage and current values of the resistor are proportional to each other.

$$U = RI \quad (42.4)$$

The R is electrical resistance .

– **I Kirchhoff's law**

The algebraic sum of currents in a node equals zero.

$$\sum_k I_k = 0 \quad (42.5)$$

– **II Kirchhoff's law**

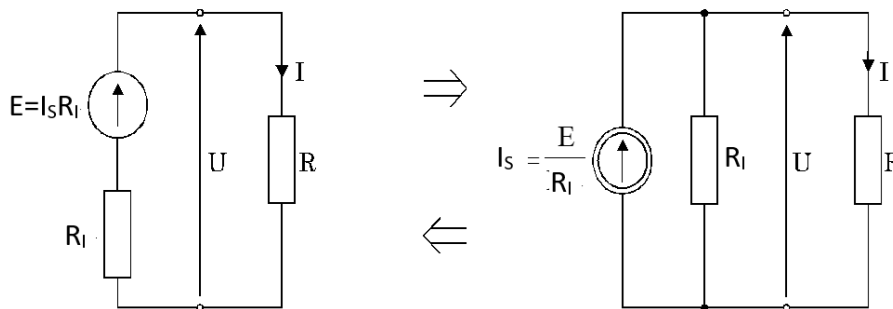
The algebraic sum of all voltages along any closed path in the electrical circuit equals zero.

$$\sum_k U_k = 0 \quad (42.6)$$

**Equivalent systems**

Two systems with the same number of terminals are called equivalent when, at the same voltages between the corresponding terminals, the same currents flow in the wires connected to these terminals. Calculation of electrical circuits can be simplified by replacing certain connections by equivalent circuits (transformation energy sources, combining resistors).

– **Transformation of energy sources**



**Figure 42.3.** Transformation of energy sources

Real source of voltage  $E$  and internal resistance  $R_I$  can be replaced by real current source  $I_S = \frac{E}{R_I}$  and internal resistance  $R_I$ .

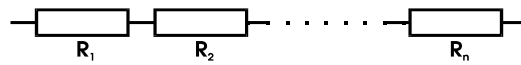
Real current source  $I_S$  with the internal resistance of  $R_I$  can be replaced by the real voltage source of the voltage  $E = I_S R_I$  and internal resistance  $R_I$ .

– **Resistors connection**

**a) Serial**

Series of resistors, through each resistor flows the same current, while the voltage on the serial connection equals the sum of the voltages on each resistor, which are different and according to Ohm's law depend on the resistance value of individual resistors. The total resistance of a serial resistors connection is given by the formula:

$$R_{eq} = \sum_{k=1}^n R_k \quad (42.7)$$

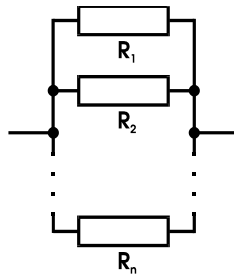


**Figure 42.4.** Replacement of resistances connected in series with equivalent resistance

**b) Parallel**

Parallel resistors, each resistor has the same voltage, while the incoming current to the connection is the sum of the currents flowing through each resistors, which are different and according to Ohm's law depend on the resistance value of individual resistors. The total resistance of the parallel resistors connection is given by the formula:

$$\frac{1}{R_{eq}} = \sum_{k=1}^n \frac{1}{R_k} \quad (42.8)$$



**Figure 42.4.** Replacement of resistances connected in parallel with equivalent resistance

**Star connection conversion to triangle and triangle connection conversion to star**



**Figure 42.5.** Star conversion to triangle and triangle conversion to star

Equations for the values of the equivalent resistances when converting a star to a triangle:

$$\star \rightarrow \Delta \quad R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3} \text{ etc...} \quad (42.9)$$

and triangle to a star:

$$\Delta \rightarrow \star \quad R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \text{ etc...} \quad (42.10)$$

**Calculations methods**

– **Kirchhoff's laws method**

The calculation of currents and voltages in the circuit can be performed with the Kirchhoff's laws.

The number of unknowns (currents or voltages)  $n$  must be equal to the number of  $n$  equations.

If the circuit has  $n$  branches and nodes  $\alpha$  it can be arranged:

$$\alpha - 1 \text{ equations from the I Kirchhoff law } \sum_k I_k = 0 \quad (42.11)$$

and the other,  $n - \alpha + 1$  equations from the II Kirchhoff law  $\sum_k U_k = 0$  (42.12).

– **Mesh current analysis**

The mesh current analysis allows for a circuit with  $n$  branches to form the  $n - \alpha + 1$  equations.

Matrix equation resulting from the mesh current analysis:

$$\mathbf{R} \cdot \mathbf{I} = \mathbf{E} \quad (42.13)$$

Where:  $\mathbf{R}$  - Rod resistance matrix,

$R_{ij}$  - Self-resistance of the mesh-total resistance in the mesh,

$R_{ij}$  - Mutual resistance of the mesh, the resistance has a + if the mesh currents in the common branch have matching phrases, and – when the opposite,

$\mathbf{E}$  - Vector of mesh voltages,

$E_i$  - The sum of the source voltages in the mesh (the components of the sum have a positive sign, when the source voltage return is consistent with the resupply of the mesh current).

– **Nodal analysis**

The nodal analysis allows for a circuit with  $n$  branches to form  $\alpha - 1$  equations.  
Matrix equation resulting from the nodal analysis:

$$\mathbf{G} \cdot \mathbf{V} = \mathbf{I}_w \quad (42.14)$$

Where:  $\mathbf{G}$  - Nodular conductivity matrix,

$G_{ii}$  - Self-conductance of the node - the sum of the conductance of the branches coincide in the node,

$G_{ij}$  - Reciprocal conductance- the sum with the sign -, the conductance of the branches joining the nodes  $i, j$



$I_w$  - Vector of nodes currents,

$I_i$  - Algebraic sum of the ratio  $\sum_i G E$  of branches that coincide with the nodes (the components of the sum have a positive sign when the source voltage is directed to the node).

– ***Superposition method***

The current in any branch of the linear circuit, with the operation of all energy sources, is the algebraic sum of all currents that flow due to the operation of each energy source separately.

Removal of the source from the circuit consists of short circuits of voltage sources and the opening of current sources.

***Theorems***

– ***Thevenin's Theorem***

Each linear active component can be represented as an real voltage source.

The source voltage of the substitute source is equal to the voltage on the two-wire terminals in the idle state, and the internal resistance of the replacement source is the resistance seen from the two-piece terminals after removal of the voltage sources (short circuit) and current sources (opening the branches).

– ***Norton's Theorem***

Each linear active component can be represented as a real current source.

The source current of the replacement current source is equal to the current flowing through the compact terminals of the two-wire, and the internal resistance of the replacement source is the resistance seen from the two-piece terminals after removal of the voltage sources (short circuit) and power sources (opening branches).

## 42.2. Laboratory tests

### 42.2.1 Ohm's law

Connect the layout **1**. on the training board (Fig.42.6). To do this connect ammeter and voltmeter to the system and freely selected resistor from the set of resistors located on the training board. Connect the voltage to the circuit number **1** and next by adjusting the voltage source potentiometer, perform measurements and plot the characteristics  $U=f(I)$  for the selected resistor. Note the measurement results in table 42.1.

**Table 42.1.**

No.	U	I	R
	V	mA	$\Omega$
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			

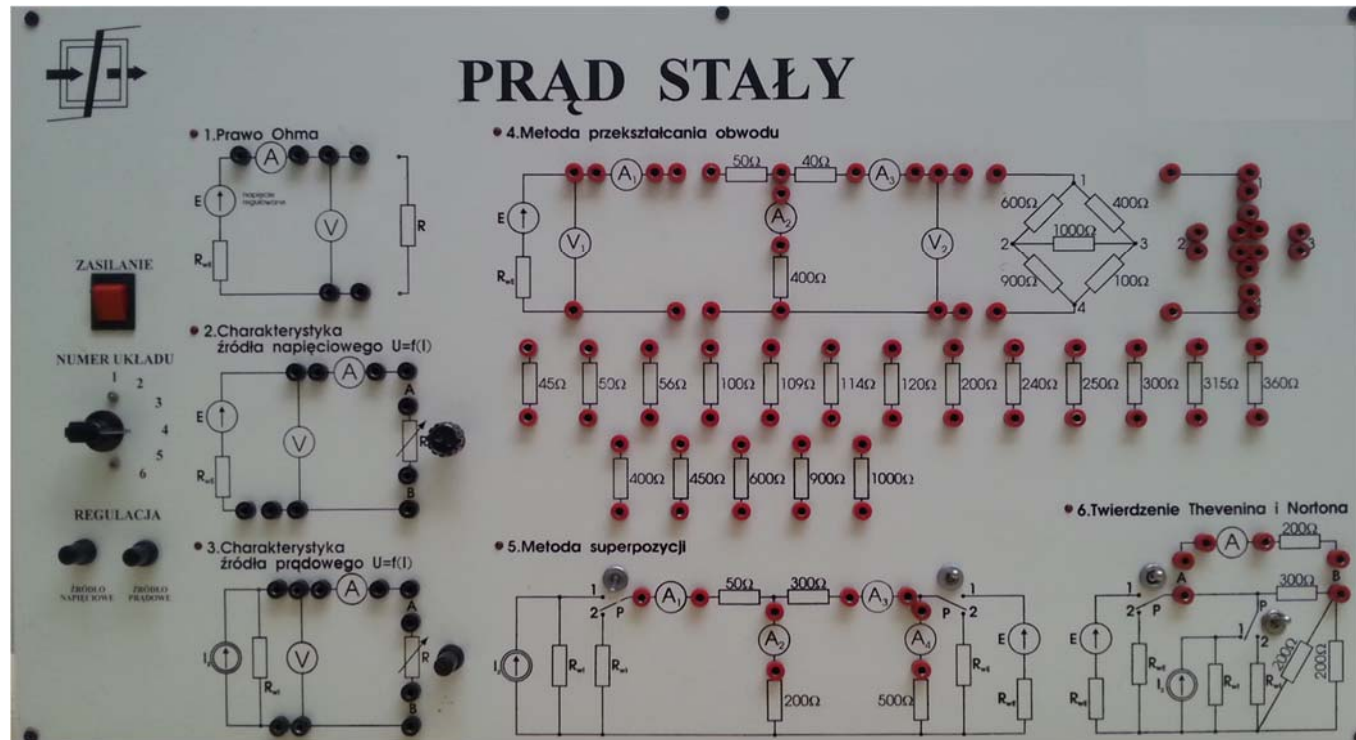


Figure 42.6. Training board

**42.2.2. Characteristics of voltage and current sources**

Connect the layout **2** on the training board (Fig. 42.6) (according to Fig. 42.2), leaving the AB terminals open. Connect voltage to the circuit **2**. By adjusting the voltage source potentiometer, set the source voltage  $E = 5\text{ V}$  or  $E = 2\text{ V}$ . Save the first point in table 2 with the current  $I$  equal to zero. Set the adjustable  $R$  resistor to the maximum value (right extreme position), connect the resistor to the voltage source AB terminals. Perform a series of measurements, reducing the resistance value from the maximum value to zero. Note the measurement results in table 42.2.

**Table 42.2.**

No.	U	I
	V	mA
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		

Connect the layout **3** on the training board (Fig. 42.6) (according to Fig. 42.2) short the terminals AB. Connect source to circuit **3**. Set the source current  $I_S = 10 \text{ mA}$  or  $I_S = 4 \text{ mA}$  by adjusting the current source potentiometer. Set the adjustable R resistor to zero (left extreme position). Connect the resistor to the current source AB terminals. Perform a series of measurements by increasing the R resistance value so as to the voltage on the source does not exceed 5V. Note the measurement results in table 42.3.

**Table 42.3.**

No.	U	I
	V	mA
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		

## 42.2.3. Kirchoff's Laws

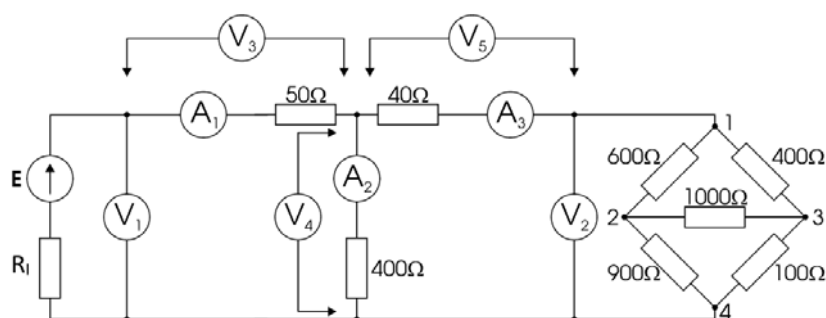


Figure 42.7. Basic test circuit

Connect the layout **4** on the training board (Fig. 42.6) (according to Fig. 42.7) connect the meters in the places of the breaks in the circuit connect the jumpers, leaving the source terminals open. Connect the voltage to the circuit **4** and adjust the voltage source by potentiometer to  $E = 5$  V. Connect the jumper connecting the source to the rest of the circuit. Check that the sum of currents in the node is equals zero. Note the results in table 42.4.

Table 42.4.

$I_1$	$I_2$	$I_3$	$\Sigma I$
mA	mA	mA	mA

Check that the sum of the voltages in the closed circuits equals zero, including the additional voltmeter (as shown in Fig. 42.6. ) on branches with resistance  $50 \Omega$ ,  $40 \Omega$  and  $400 \Omega$ . Note the results in table 42.5.

Table 42.5 .

					Mesh 1	Mesh 2
$U_1$	$U_3$	$U_4$	$U_5$	$U_2$	$\Sigma U$	$\Sigma U$
V	V	V	V	V	V	V

#### 42.2.4. Method of the circuit transformation

This part of the laboratory studies consists of replacement of the system of resistors with a simpler equivalent system and performing measurements after each change in the system. The results of the measurements shall be noted in table 42.6.

- Perform measurements in the basic layout (Fig. 42.6.)
- Transform part of the circuit **1-4** to equivalent circuit (Fig. 42.6)
  - convert  $\Delta \rightarrow \text{Y}$ 

Perform the calculations and assemble the new circuit according to the calculation (using special terminals on the plate and a set of resistors). Detach the basic **1-4** layout and connect the equivalent circuit that is assembled by it. Perform measurements.
  - Serial-parallel connection
 

Calculate the equivalent resistance of the serial-parallel connection and, as calculated, assemble and connect the new circuit. Perform measurements.
  - Serial connection
 

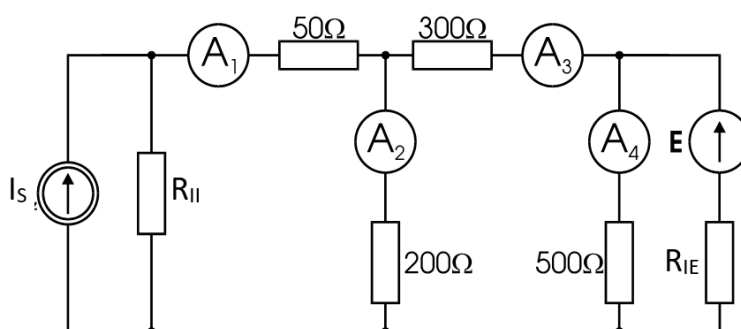
Replace circuit **1-4** with calculated equivalent resistance. Perform measurements.
- Calculate the equivalent resistance of the entire circuit

Replace the entire circuit connected to the source terminals by the resistor calculated by yourself (parallel and serial connections). Values of current  $I_1$  and voltage on source  $U_1$  note in table 42.6.

**Table 42.6.**

	$U_1$	$I_1$	$I_2$	$I_3$	$U_2$
	V	mA	ma	mA	V
Primary circuit					
1 equivalent circuit					
2 equivalent circuit					
3 equivalent circuit					

#### 42.2.5. Superposition method



**Figure. 42.8.** Measuring system with two energy sources to validate the application of the superposition method



- Set the source voltage  $E = 5\text{ V}$  and source current  $I_S = 10\text{ mA}$ . If the positions of voltage and current sources potentiometers are changed:
  - Connect the voltmeter to system **2** (Fig. 42.6) (according to Fig. 42.1) on the training board, with the AB terminals and adjusting the voltage source potentiometer, set the source voltage  $E = 5\text{ V}$ ,
  - Connect on the ammeter to the system **3** (Fig. 42.6) on the training board (according to Fig. 42.2), with short AB terminals and adjusting the current source potentiometer, set the source current  $I_S = 10\text{ mA}$ ,

then

- Connect the meters into system **5** on the training board (Fig. 42.6),
- Set the p-switches to position **1**,
- Connect sources to system **5**.

Perform measurements and note the results in table 42.7.

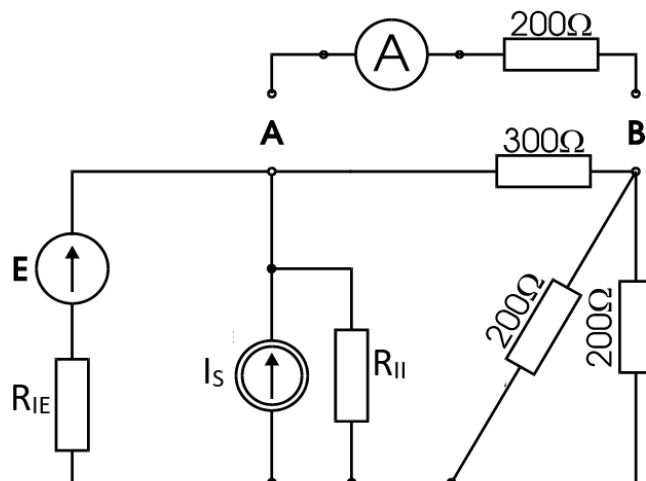
**Table 42.7.**

Connected sources:	$I_1$	$I_2$	$I_3$	$I_4$
	mA	mA	mA	mA
Both simultaneously				
Only voltage				
Only current				
$\Sigma I$				

- Open the current source, leaving its internal resistance (Fig.42.6- switch p at current source set in position **2**). Perform the measurements, note the results in table 42.7.

- Connect the current source (Fig. 42.6 switch p at current source set to position **1**)
- Short voltage source leaving its internal resistance (Fig.42.6 switch with voltage source in position **2**). Perform the measurements, note the results in table 42.7.

#### 42.2.6. Thevenin's and Norton's Theorem



**Figure 42.9.** Measuring system with two energy sources to verify the correctness of Thevenin's and Norton's Theorem

- Set the source voltage  $E = 2 \text{ V}$  and source current  $I_S = 4 \text{ mA}$ . To do it , proceed as described in point 2.5.
- Set the switches to **1** Fig. 42.6.
- To the terminals AB connect the branch with resistance  $200 \Omega$  and ammeter.
- Connect the sources to the system and measure the current  $I$ . The measurement results note in table 42.9.
- Open the AB branch.
- Measure by voltmeter the voltage on the AB terminals in the idle state  $U_{AB}$ .
- Short the AB terminals with the ammeter and measure the short-circuit current  $I_{SAB}$ .

- Turn off the sources from layout 6 on the training board (Fig.4 2.6) (according to Fig 42.9).
- Measure the resistance of the system  $R_{AB}$  as seen from the AB terminals.
- Note the measurement results in table 42.8.

**Table 42.8.**

	$U_{AB}$	$I_{SAB}$	$R_{AB}$
	V	mA	$\Omega$
Measurement			
Calculations			

- To layout **2** on the training board (Fig. 42.6) in series with  $R_{IE}$  enable such resistor so that the created voltage source has a resistance equal to the resistance of  $R_{AB}$  measured in system **6** on the training board (Fig. 42.6) (according to Fig. 42.9). Connect the voltmeter, leave the AB terminals open. Connect source to system **2** (Fig. 42.6). Use potentiometer to set the source voltage equal to the voltage  $U_{AB}$ .
- AB branch with ammeter and  $200 \Omega$  resistor (from **6** layout – Fig. 42.6) connect the actual voltage source with the internal resistance  $R_{AB}$  and the source voltage  $U_{AB}$ .
- Measure the current flowing through the connected branch. Note the result in table 42.9.
- To system **3** on the training board connect such resistance in parallel, so that the current source has a resistance equal to the resistance of  $R_{AB}$  measured in system **6**. Connect the ammeter and short the AB terminals. Connect the source to the system **3**. Use the potentiometer to set the source current equal  $I_{SAB}$ .

- Connect the AB branch of layout 6 on the training board to the AB terminals of the actual current source with the internal resistance  $R_{AB}$  and the source current equal to  $I_{sAB}$ . Measure the current flowing through the connected branch. The measurement result note in table 42.9.

Table 42.9.

Current measurement in layout 6	Thevenin's Theorem		Norton's Theorem	
	Calculations	Measurement	Calculations	Measurement
mA	mA	mA	mA	mA

The report shall include:

- Plotted on millimeter paper the characteristics of the tested resistor  $U = f(I)$ ,
- Calculate the resistance value for several points and compare it to the value stated on the plate,
- Plot the characteristic of the voltage source  $U = f(I)$ ,
- Graphically determine the value of the short-circuit current of the voltage source,
- Calculated on the basis of the characteristics of the voltage source, its internal resistance  $R_{IE}$ ,
- Plot the characteristics of the current source  $U = f(I)$ ,
- By extending the resulting straight to the U axis, graphically designate the voltage value at the current source  $I_s R_{II}$
- Calculated on the basis of the current source characteristics, its internal resistance  $R_{II}$

- Provide a review of Kirchhoff's laws,
- Compare the results obtained during the successive phases of the circuit transformation,
- Compare, resulting from the superposition method, the sum of algebraic currents in the branches at the action of each source separately, with the currents flowing in the branches, when both sources operate simultaneously,
- Using measured values  $U_{AB}$ ,  $I_{AB}$  and  $R_{AB}$  - present the active component AB in the form of a real voltage source (Thevenin's Theorem) and the real current source (Norton's Theorem) and calculate the current in the branch with resistor  $200 \Omega$  (note: the ammeter resistance must be taken into account),
- Compare the current calculated on the basis of both theorems with the measurement results,
- Calculate the values of  $U_{AB}$ ,  $I_{SAB}$  and  $R_{AB}$  based on layout 6 data and compare with the measured values.

### 42.3. Remarks and conclusions

Assess the correctness of the use of laws, theorems and computational methods in DC circuits.

#### References:

- [1]. Krakowski M.: Theoretical electrical Engineering. Vol. 1: line and non-linear circuits, WNT, Warsaw-Poznań 1995
- [2]. Kurdziel R.: Fundamentals of Electrical Engineering, WNT, Warsaw 1972