EXERCISE 50
TEST OF TRANSIENT STATES

The purpose of the laboratiry study.

The purpose of this exercise is to learn the phenomenon of transient states in electrical circuits containing resistor (R), capacitor (C) and inductor (L).

I. General information

1. Introduction

In the case of DC generators, the voltages and currents in the circuits are constans quatities, and in the case of sinusoidal current generators, voltages and currents change sinusoidally. This type of circuit state is called stady or stationary state. In electrical circuits there is also a phenomena caused by a change in the circuit, such as switching the power source into a circuit or shorting a part of the circuit. These phenomena are called transition or transient states. In general, transient states are defined as physical processes occurring at transformation an electrical circuit from one steady state to another. In many cases, transient states are undesirable phenomena. For example, transient phenomena occurring at short circuits and when voltages are switched on in electrical circuits.
In other cases, transient states are the normal operating state of devices, e.g. automatic adjustment systems.

In the analysis of the transient states of the electrical circuits, the voltage u and current i are presented in the form of a sum of two components:

$$u=u_u+u_p \tag{1}$$

$$i=i_u+i_p. \tag{2}$$

The u_u i i_u quantities are staedy state components of voltages and currents, while u_p and i_p are the transient components of these quantities.

Any disruption of the system causes the transient state. The physical system is called stable when it returns to the equilibrium state after the disruption. When the system is stable, transient states disappear after a sufficient period of time. Therefore, in a stable system the transient components disappear over time, i.e. the

$u_p \to 0$ and $i_p \to 0$, when $t \to \infty$.

It follows that in stable systems

$u \to u_u$ and $i \to i_u$, where $t \to \infty$,

so, after sufficient time, a steady state is occurred in the system.

Theoretically, the transient state takes infinitely long time, but practically after a sufficient period of time the circuit achieves a steady state.

To facilitate the analysis of transient states, it is assumed that a disturbance that is a source of transient occurred at $t = 0$.This is the initial state.
The values of variables in the initial state are called initial conditions.

Essential features of electrical systems containing R, L and C components, are two conditions resulting from the principle of energy conservation (inductor current and voltage on capacitor continuity conditions):

1) the current in the inductor must change continuously; if it had changed in a stepped way, the inductor would induce infinitely high voltage resulting from the formula (28), which is impossible,
2) the voltage on the capacitor must change continuously; if it changed in a stepped way, the capacitor would flow infinitely high current resulting form the formula (6), which is impossible.

These conditions can be written as follows:

ad 1) $i(0^-)=i(0)=i(0^+)$ (3)

ad 2) $u_C(0^-)=u_C(0)=u_C(0^+)$ (4)

where $i(0^-)$ and $i(0^+)$ mean respectively the left and right-hand limits of the function $i(t)$ at time $t = 0$,. and $u_C(0^-)$ and $u_C(0^+)$ means the left and right limits of the function $u_C(t)$ in thet ime $t = 0$.
Considering state of the circuit immediately before t = 0 in which the disruption occurred, we designate the current $i(0^-)$ in the inductor and the voltage $u_C(0^-)$ on the capacitor.
The initial values $i(0),u_C(0)$ are equal to $i(0^-)$, $u_C(0^-)$.
Considering all the inductors and capacitors in the circuit, a sufficient number of initial conditions are obtained, necessary to solve the differential equations.

2. RC Circuits

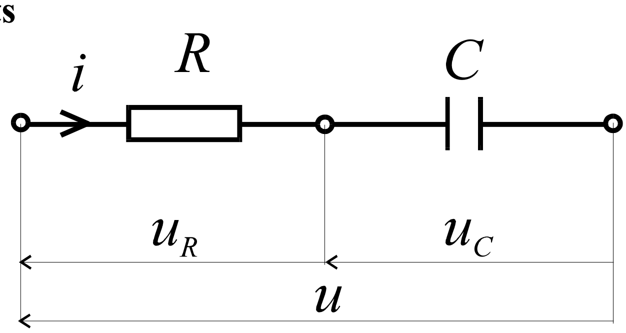


Fig. 1. Serial connection of R, C elements.

The voltage at the terminals of serial connection of the R, C (F*ig. 1*) is expressed by formula:

$$R \cdot i + u_C = u \tag{5}$$

wherein the i – current in the circuit, u - supply voltage,
u_C -voltage on capacitor, R -resistance of the circuit.
Substituting to the above formula the expression of the current in the capacitor:

$$i = C \cdot \frac{du_C}{dt} \tag{6}$$

We receive the differential equation:

$$R \cdot C \cdot \frac{du_C}{dt} + u_C = u \tag{7}$$

The simplified differential equation is expressed by the formula:

$$R \cdot C \cdot \frac{du_{Cp}}{dt} + u_{cp} = 0 \tag{8}$$

The general solution of the above simplified equation is the equation called **the Helmholtz equation**:

$$u_{Cp} = A \cdot e^{-t/\tau} \tag{9}$$

wherein A -constant, and $\tau = RC$ is the time constant of the RC circuit.
The inverse of the time constant τ are called the damping constant $\alpha = 1/RC$.

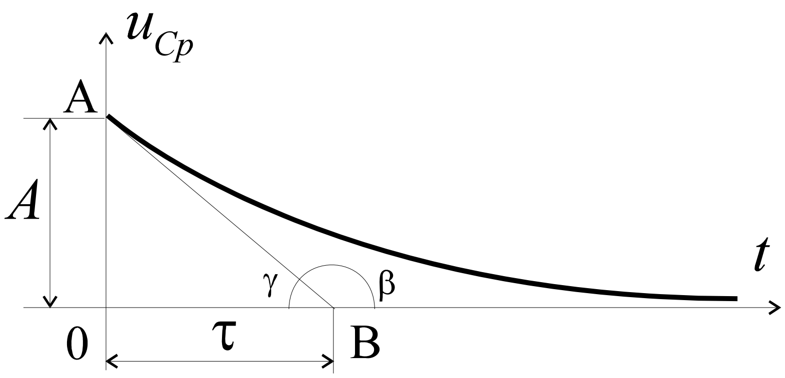


Fig. 2. Course of transient voltage on capacitor in a series of R, C elements.

A time constant equal to the subtangent **OB,** characterized the speed of decreasing the transient component u_{Cp} (*Fig. 2*). Time constant τ is the time after which the voltage u_{Cp} would have achieved a value equal to zero if the speed of its reduction was constant and equal to the speed at the moment $t = 0$, so $\left(\frac{du_{Cp}}{dt}\right)_{t=0} = -\frac{A}{RC}$.

When the time constant is low (large damping), then the exponential curve is steep, therefore the voltage u_{Cp} decreases rapidly.
If the time constant is high (small damping), then the exponential curve is flat, therefore voltage u_{Cp} decreases relatively slowly.

2.1. Switching on the DC voltage

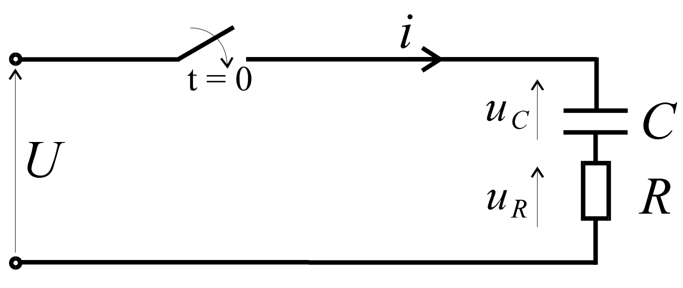


Fig. 3. Scheme of DC voltage charging system of capacitor.

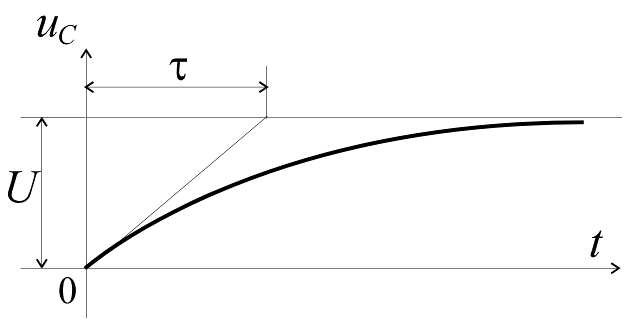


Fig. 4. The voltage waveform on the capacitor in the system from Fig. 3.

When closing switch at t = 0, the DC voltage is connected to the circuit (F*ig. 3*). After sufficient time, the capacitor *C* charge to voltage *U*, therefore the voltage is determined $u_{Cu} = U$. According to the formula (1) and (9), we get a relationship to the instantaneous value of the voltage on the capacitor:

$$u_C = u_{Cu} + u_{Cp} = U + A \cdot e^{-t/\tau} \tag{10}$$

the time constant of the considering circuit is $\tau = RC$.
Assuming that the capacitor C was not charge before closing the switch, we have $u_C(0^-) = 0$, according to the equation (4) the voltage on the capacitor $u_C(0) = 0$.
Substituting t = 0 to the formula (10), we get $U + A = 0$, so $A = -U$.

The instantaneous value of the voltage on the capacitor (*fFig. 4*) is expressed by the formula:

$$u_C = U \cdot (1 - e^{-t/\tau}). \tag{11}$$

Voltage on the resistor

$$u_R = U - u_C = U \cdot e^{-t/\tau} \tag{12}$$

decreases exponentially from *U* to 0 with time constant τ. Current in the considering circuit:

$$i = \frac{u_R}{R} = \frac{U}{R} e^{-t/\tau} \tag{13}$$

exponentially disappears from *U/R* to 0 with time constant τ..

2.2. Discharge of the capacitor

The capacitor *C* (*Fig. 5*) has been initially charged to the *U* voltage.
At the moment of t = 0, we short the capacitor by switch through the resistor, which means the capacitor is discharged.
After a sufficient period of time from t = 0, the capacitor will be discharged, wherein the voltage on it decreases to zero, therefore the establisehd voltage on the capacitor $u_{Cu} = 0$.

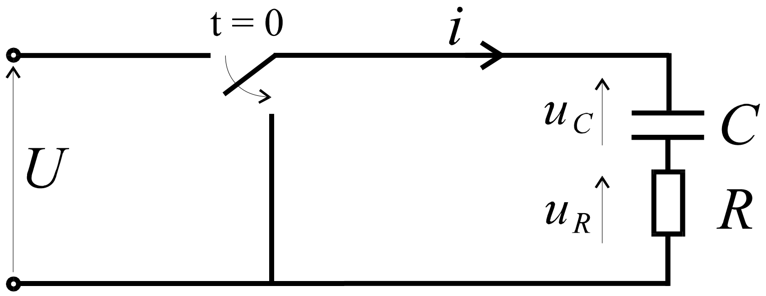


Fig. 5. Scheme of capacitor discharge system.

The instantaneous value of the capacitor voltage is expressed by equation:

$$u_C = u_{Cu} + u_{Cp} = A \cdot e^{-t/\tau}, \tag{14}$$

wherein the time constant $\tau = RC$.
Directly before switching the switch , the voltage on the capacitor was equal

$$u_C(0^-) = u_C(0) = U. \tag{15}$$

Substituting t = 0 to (14), we receive $A = U$, therefore $u_C = U \cdot e^{-t/\tau}$.
The voltage on the capacitor disappears exponentially from U to 0 with time constant τ. The current in the circuit can be determined from the equation (6):

$$i = C\frac{du_C}{dt} = C \cdot U \cdot e^{-t/\tau} \cdot \left(-\frac{1}{\tau}\right) = -\frac{U}{R} \cdot e^{-t/\tau}. \tag{16}$$

The energy of the capacitor electrical field before switching was $W_C = \frac{1}{2} \cdot C \cdot U^2$

After the phenomena have been established in the circuit, the capacitor is discharged, so its energy equals zero. During discharging the capacitor, the current is flowing expressed by formula (16), so in the resistor R a conversion of electricity into heat is occurred. The power lost in the R resistor is

$$p_R = R \cdot i^2 = \frac{U^2}{R} \cdot e^{-2 \cdot t/\tau}. \tag{17}$$

The thermal energy produced in the resistor equals

$$W_R = \int_0^\infty p_R dt = \frac{U^2}{R} \cdot \int_0^\infty e^{-2 \cdot t/\tau} dt = \frac{1}{2} \cdot C \cdot U^2 = W_C. \tag{18}$$

This means that the energy of the capacitor electric field is completely transformed into Joule's heat in the R resistor.

2.3. Switching on sine wave voltage

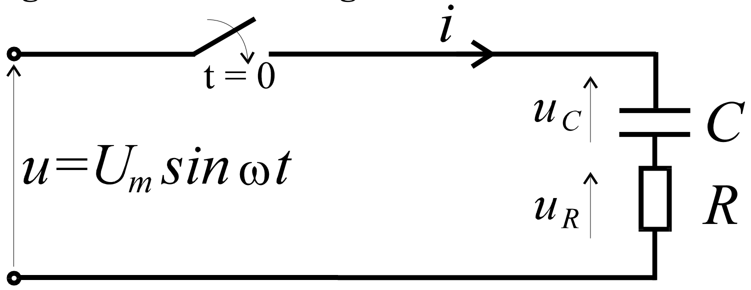


Fig. 6. Switching on sine wave voltage to the R, C connection.

Closing switch in time t = 0, to the circuit (*Fig. 6*) sinusoidally variable voltage is switched on

$$u = U_m \sin(\omega t + \varphi_u). \tag{19}$$

After a sufficient period of time from the moment t = 0 in the circuit flows a steady current

$$i_u = \frac{U_m}{Z}\sin(\omega t + \varphi_u - \varphi), \tag{20}$$

where

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}, \quad tg\varphi = -\frac{1}{\omega CR}. \tag{21}$$

The established voltage u_{Cu} on the capacitor is delayed by 90° to the current i_u,

$$u_{Cu} = \frac{U_m}{Z\omega C}\cdot \sin(\omega t + \varphi_u - \varphi - 90^\circ) = -\frac{U_m}{Z\omega C}\cdot \cos(\omega t + \varphi_u - \varphi). \tag{22}$$

Temporary value of the voltage on the capacitor

$$u_C = u_{Cu} + u_{Cp} = -\frac{U_m}{Z\omega C}\cdot \cos(\omega t + \varphi_u - \varphi) + A\cdot e^{-t/\tau} \tag{23}$$

where $\tau = RC$.

Suppose that the capacitor was not charged before closing the switch, i.e. the $u_C(0^-) = u_C(0) = 0$. Substituting t = 0 to the formula (23), we receive

$$-\frac{U_m}{Z\omega C}\cdot \cos(\varphi_u - \varphi) + A = 0, \text{ so} \qquad A = \frac{U_m}{Z\omega C}\cdot \cos(\varphi_u - \varphi). \tag{24}$$

Temporary value of the voltage on the capacitor

$$u_C = \frac{U_m}{Z\omega C}\cdot \left[-\cos(\omega t + \varphi_u - \varphi) + e^{-t/\tau}\cdot \cos(\varphi_u - \varphi)\right]. \tag{25}$$

The temporary current value in the circuit

$$i = C\frac{du_C}{dt} = \frac{U_m}{Z}\cdot \left[\sin(\omega t + \varphi_u - \varphi) - \frac{e^{-t/\tau}}{\omega CR}\cdot \cos(\varphi_u - \varphi)\right]. \tag{26}$$

3. RL circuits

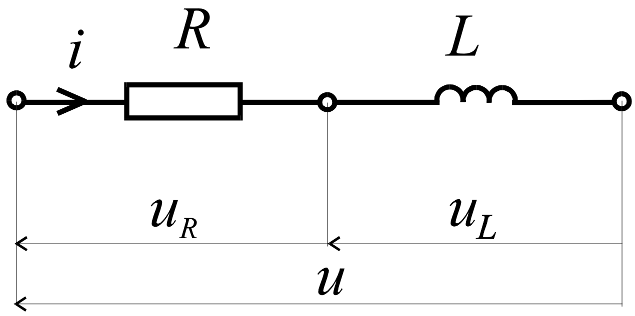


Fig. 7. Serial connection of R, L elements.

The voltage on the serial connection of the *R, L* elements is expressed by:

$$R \cdot i + u_L = u \tag{27}$$

where *i* -current in the circuit, *u* - supply voltage,
R- the resistance of the circuit, u_L -voltage on the inductor expressed by the formula:

$$u_L = L\frac{di}{dt} \tag{28}$$

where *L* -inductance of the circuit.
The simplified differential equation for transient current takes the form of

$$R \cdot i_p + L\frac{di_p}{dt} = 0 \tag{29}$$

where i_p -component of the transient current,
other markings as before.

The general solution of the above simplified equation is the equation called **the Helmholtz equation**:

$$i_p = A \cdot e^{-t/\tau} \tag{30}$$

wherein *A* -constant, and $\tau = L/R$ is the time constant of the RL circuit.
The inverse of the time constant τ are called the damping constant $\alpha = L/R$.

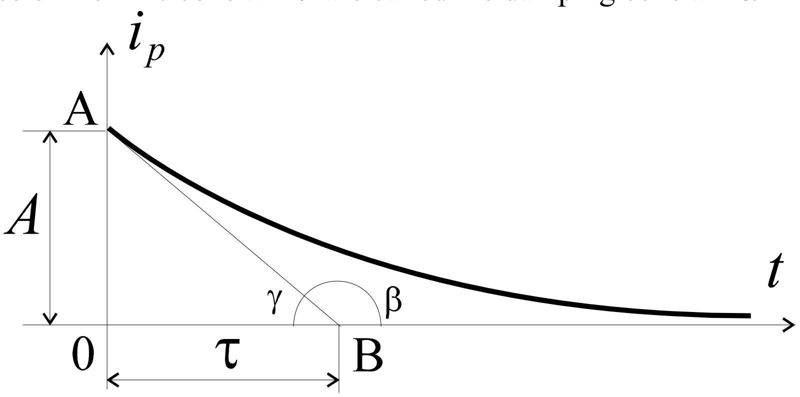


Fig. 8. Course of transient current in connected R, L elements.

The time constant is characterized the speed of decreasing current i_p (*Fig. 8*).
When the time constant is low (large damping), then the exponential curve is steep, so the magnitude i_p decreases quickly.
If the time constant is high (small damping), then the exponential curve is flat, so the magnitude i_p decreases relatively slowly.
When $t > 5\tau$, then $e^{-t/\tau} < 0.01$, the transient current i_p become negligible and the total current achieve the determined value.

3.1. Switching on the DC voltage

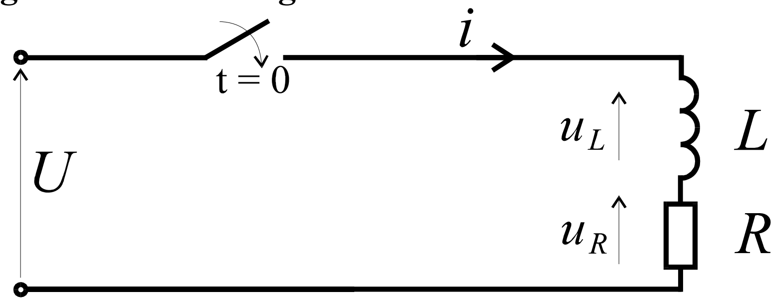


Fig. 9. Switching on the DC voltage to the R, L circuit.

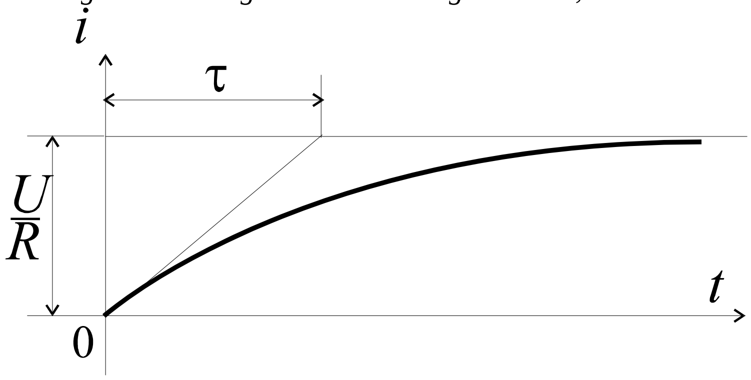


Fig. 10. Current waveform on branch with R, L in circuit from Fig. 9.

Closing switch at time t = 0, the DC voltage is connected to the circuit (*Fig. 9*)
After sufficient time from t = 0, in the circuit flows the current

$$i_u = \frac{U}{R}. \tag{31}$$

According to the formulas (2) and (30), we get a dependency on the temporary value of the inductor:

$$i = i_u + i_p = \frac{U}{R} + A \cdot e^{-t/\tau} \tag{32}$$

The time constant of the considering circuit is $\tau = L/R$.

Directly before the switch on, the circuit was interrupted, so $i(0^-) = 0$, and according to the equation (3) current in the circuit $i(0) = 0$.

Substituting t = 0 for the formula (32), we get $\frac{U}{R} + A = 0$, and therefore $A = -\frac{U}{R}$.

The temporary value of the current in the concidering circuit (*Fig. 9*) is expressed by the formula:

$$i = \frac{U}{R} \cdot (1 - e^{-t/\tau}). \tag{33}$$

Voltage on the resistor

$$u_R = R \cdot i = U\left(1 - e^{-t/\tau}\right) \tag{34}$$

exponentially increases from 0 to U with time constant τ.

Voltage on the inductor

$$u_L = U - u_R = U \cdot e^{-t/\tau} \tag{35}$$

decreases exponentially from U to 0 with time constant τ (voltage u_L can also be determined from equation (28)).

3.2. Short circuit through inductor

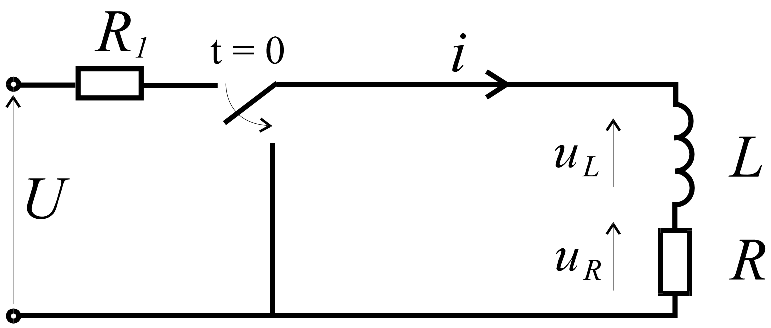


Fig. 11. Shorting branches with R, L elements.

At the moment t = 0 we short a branch of the circuit containing the elements R and L. After a sufficiently long period of time from t = 0, the current in this branch will decrease to zero, so the value is determined $i_u = 0$.
The temporary value of current in the induction inductor is expressed by

$$i = i_u + i_p = A \cdot e^{-t/\tau}, \tag{36}$$

wherein the time constant $\tau = L/R$.

Directly before switching in the circuit, the DC current flowed $\frac{U}{R_1 + R}$

$$i(0^-) = i(0) = \frac{U}{R_1 + R}. \tag{37}$$

Substituting t = 0 for the equation (36), we receive $A = \frac{U}{R_1 + R}$, therefore,

$$i = \frac{U}{R_1 + R} \cdot e^{-t/\tau} . \tag{38}$$

The current in a shorten branch disappears exponentially from $\frac{U}{R_1 + R}$ to 0 with the time constant τ.

Let's say that the circuit containing the inductor flows current $I_o = \frac{U}{R_1 + R}$, so the inductor energy contained in its magnetic field is $W_L = \frac{1}{2} \cdot L \cdot I_o{}^2$.

After the phenomena are established in the circuit, the current in the inductor is equal to zero, so its energy is also equal to zero.

Due to the fact that during the short circuit of the RL the current is flowing (equation 38), then in the resistance *R* of the inductor decline a conversion of electrical energy into heat.

The power resulting from the inductor resistance is

$$P_R = R \cdot i^2 = R \cdot I_o^2 \cdot e^{-2 \cdot t/\tau} . \tag{39}$$

The thermal energy resulting from the inductor resistance equals

$$W_R = \int_0^\infty p_R dt = R \cdot I_o^2 \cdot \int_0^\infty e^{-2 \cdot t/\tau} dt = \frac{1}{2} \cdot L \cdot I_o^2 = W_L . \tag{40}$$

This means that during the transient state period all the energy W_L contained in the magnetic field of the inductor is converted to joule heat (in inductor resistance).

3.3. Switching on sine wave voltage

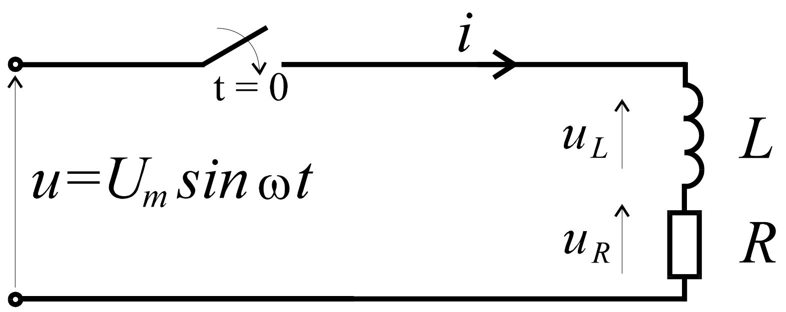


Fig. 12. Switching on the sine wave voltage to the R, L connection.

Closing switch in time t = 0, to the circuit (*Fig. 12*) sinusoidally variable voltage is switched on

$$u = U_m \sin(\omega t + \varphi_u) . \tag{19}$$

After a sufficient period of time from the moment t = 0 in the circuit flows a steady current

$$i_u = \frac{U_m}{Z} \sin(\omega t + \varphi_u - \varphi) , \tag{20}$$

where

$$Z = \sqrt{R^2 + \omega^2 L^2}\,, \quad tg\varphi = \frac{\omega L}{R}\ . \qquad (21)$$

(43)

Temporary current value in the circuit

$$i = i_u + i_p = \frac{U_m}{Z} \cdot \sin(\omega t + \varphi_u - \varphi) + A \cdot e^{-t/\tau}\ . \qquad (44)$$

wherein τ = *L/R.*

Suppose that before closing the current in the circuit has not flowed, so $i(0^-) = i(0) = 0$

.

Substituting t = 0 to the formula (44), we receive

$$\frac{U_m}{Z} \cdot \sin(\varphi_u - \varphi) + A = 0, \qquad (45)$$

that is

$$A = -\frac{U_m}{Z} \cdot \sin(\varphi_u - \varphi)\,. \qquad (46)$$

Temporary current value in the circuit

$$i = \frac{U_m}{Z} \cdot \left[\sin(\omega t + \varphi_u - \varphi) - e^{-t/\tau} \cdot \sin(\varphi_u - \varphi)\right]. \qquad (47)$$

4. RLC Circuits

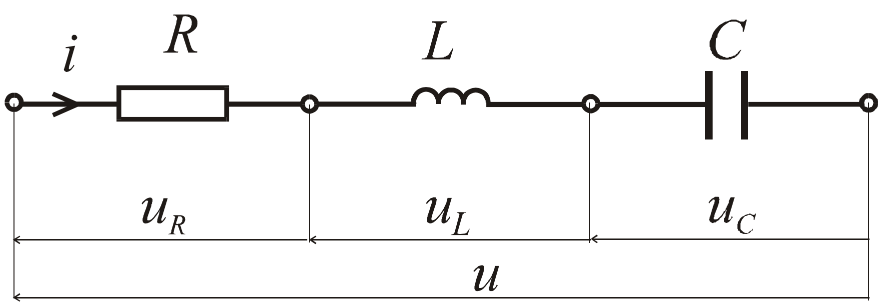


Fig. 13. Serial connection of R, L, C elements.

The voltage on the serial connections of the R, L, C elements is expressed as follows:

$$R \cdot i + L\frac{di}{dt} + u_C = u \qquad (48)$$

where: *i* - current value, *u* -supply voltage, u_C -voltage on the capacitor and *R* i *L* - respectively circuit resistance and inductance.

Substituting to the above equation the formulas:

$$i = C\frac{du_C}{dt} \text{ and } \frac{di}{dt} = C\frac{d^2u_C}{dt^2} \qquad (49)$$

We receive the differential equation of the second row:

$$LC\frac{d^2u_C}{dt^2} + RC\frac{du_C}{dt} + u_C = u \tag{50}$$

The simplified differential equation is expressed by the formula:

$$LC\frac{d^2u_{Cp}}{dt^2} + RC\frac{du_{Cp}}{dt} + u_{Cp} = 0 \tag{51}$$

The characterising equation of the above differential equation is:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \tag{52}$$

and the elements of this equation are equal:

$$s_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

and (53)

$$s_2 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}};$$

Entering the markings:

α - damping constant $$\alpha = \frac{R}{2 \cdot L} \tag{54}$$

ω - pulsation of non-dampened vibration $$\omega = \frac{1}{\sqrt{LC}} \tag{55}$$

ω_0 - own circuit pulsation $$\omega_o = \sqrt{\omega^2 - \alpha^2} \tag{56}$$

In the RLC circuits can be extract three cases of circuits depending on the distinctive sign of the characteristic equation::

1) Overdamped circuit - when a relationship occurs

$$R > 2\sqrt{\frac{L}{C}} \tag{57}$$

The characteristic equation has two (different) negative elements real s_1 and s_2 ($s_1 < 0$, $s_2 < 0$ and $s_1 \neq s_2$);

2) Critically damped circuit - when the condition is fulfilled

$$R = 2\sqrt{\frac{L}{C}} \tag{58}$$

in this case there is one negative real element $s_1 = s_2 = -\alpha$;

3) Underdamped circuit - exists when

$$R < 2\sqrt{\frac{L}{C}} \tag{59}$$

In this case, the characteristic equation has two distinct complex elements $s_1 = -\alpha + j\omega_o$ and $s_2 = -\alpha - j\omega_o$.

4.1. Switching on the DC voltage

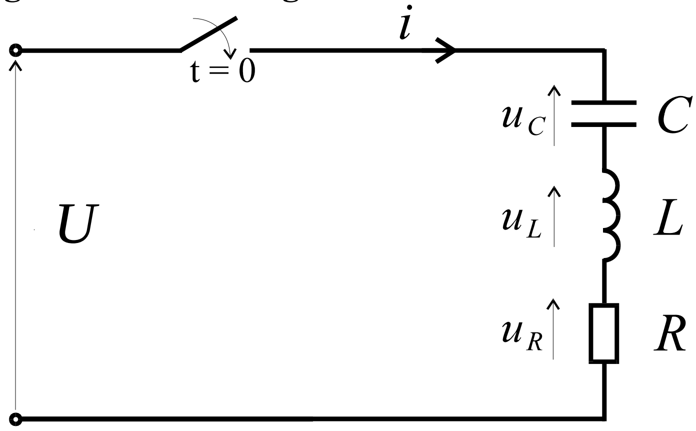


Fig. 14. Switching on the DC voltage in the circuit containing the elements R, L, C.

Closing the switch at the moment t = 0, we turn on the voltage *U* to the serial connection R, L, C (*Fig. 14*), so the capacitor is charged by the resistor and the inductor. We assume that t = 0 has not been charged. After a sufficient period of time, the capacitor is charged to the voltage *U* and then the current in the circuit will be equal to zero, that is $u_{Cu} = U$ and $i_u = 0$, therefore

$$u_C = U + u_{Cp}, \quad i = i_p . \tag{60}$$

Before the switch was closed, the capacitor voltage was equal to zero, and the current in the circuit equals zero, so on the basis of the continuity equations (3) and (4) we have:

$$i(0^-)=i(0)=0 , \quad u_C(0^-)=u_C(0)=0 \tag{61}$$

4.1.1. Overdamped circuit

$$\begin{aligned} u_{Cp} &= A_1 \cdot e^{s_1 \cdot t} + A_2 \cdot e^{s_2 \cdot t} \\ i_p &= C \cdot \left(A_1 \cdot s_1 \cdot e^{s_1 \cdot t} + A_2 \cdot s_2 \cdot e^{s_2 \cdot t} \right) \end{aligned} \tag{62}$$

where $A_1; A_2$ are integration constants. The characteristic equation (52) has two distinct negative real elements s_1 and s_2.

Substituting t = 0 to the above equation and taking into account the initial conditions specified by the formula (61) we find

$$U + A_1 + A_2 = 0 \text{, and } A_1 \cdot s_1 + A_2 \cdot s_2 = 0 . \tag{63}$$

By solving the above system of equations with two unknowhows, we find

$$A_1 = \frac{U \cdot s_2}{s_1 - s_2}, \; A_2 = -\frac{U \cdot s_1}{s_1 - s_2}, \tag{64}$$

than:

$$u_C = U \cdot \left[1 + \frac{1}{s_1 - s_2} \cdot \left(s_2 \cdot e^{s_1 t} - s_1 \cdot e^{s_2 t} \right) \right], \tag{65}$$

$$i = C \frac{du_C}{dt} = CU \frac{s_1 s_2}{(s_1 - s_2)} \left(e^{s_1 t} - e^{s_2 t} \right).$$

4.1.2. Critically damped circuit

The solution of the simplified differential equation is:

$$u_{Cp} = (A_1 + A_2 \cdot t) \cdot e^{-\alpha \cdot t} \tag{66}$$

$$i_p = C \cdot [A_2 - \alpha \cdot (A_1 + A_2 \cdot t)] \cdot e^{-\alpha \cdot t}$$

where A_1, A_2 are integration constants.
The characteristic equation of the above case has one negative element $s_1 = s_2 = -\alpha$.
Substituting t = 0 to the above equations and using the initial conditions (61), we have

$$U + A_1 = 0 \text{, and } A_2 - \alpha A_1 = 0 \tag{67}$$

so

$$A_1 = -U \text{ , } A_2 = -\alpha \cdot U \text{ .} \tag{68}$$

After substituting A_1, A_2 to the equation which are specifying u_C and i, we receive

$$u_C = U \cdot \left[1 - (1 + \alpha \cdot t) \cdot e^{-\alpha \cdot t}\right],$$

$$i = C \frac{du_C}{dt} = CU\alpha^2 t \cdot e^{-\alpha \cdot t} \text{ .} \tag{69}$$

The wave coures of the transient currents and voltages in these circuits have the same character as in the aperitic circuits. The circuit damping facor defined as the ratio of constant damping to the pulsation of the non-attenuated circuit is assumed for this case a value of 1, and for aperitic circuits this value is always greater than the one.

4.1.3. Underdamped circuit

There are two different complex elements of characteristic equation:

$$s_1 = -\alpha + j\omega_o \text{ and } s_1 = -\alpha - j\omega_o \text{ ;} \tag{70}$$

where α - constant damping $\qquad \alpha = \frac{R}{2 \cdot L}$ (71)

ω - pulsation of non-dampened vibration $\qquad \omega = \frac{1}{\sqrt{LC}}$ (72)

ω_o - own circuit pulsation $\qquad \omega_o = \sqrt{\omega^2 - \alpha^2}$ (73)

The solution of the simplified differential equation for the case under consideration is:

$$u_{Cp} = A \cdot e^{-\alpha \cdot t} \sin(\omega_o \cdot t + \beta) \tag{74}$$

$$i_p = C \cdot A \cdot \left[-\alpha \cdot e^{-\alpha \cdot t} \cdot \sin(\omega_o \cdot t + \beta) + \omega_o \cdot e^{-\alpha \cdot t} \cdot \cos(\omega_o \cdot t + \beta)\right]$$

where A, $ß$ are integration constans.

By performing an analysis similar to the previous cases, we receive

$$u_C = U \cdot \left[1 - \frac{1}{\omega_o \sqrt{LC}} \cdot e^{-\alpha \cdot t} \cdot \cos\left(\omega_o \cdot t + \beta - \frac{\pi}{2} \right) \right],$$

and (75)

$$i = \frac{U}{\omega_o L} \cdot e^{-\alpha \cdot t} \cdot \sin \omega_o t .$$

4.2. Switching on sine wave voltage

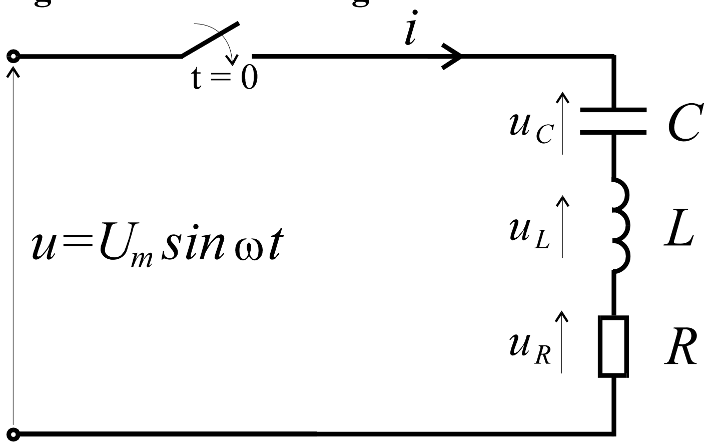


Fig. 15. Swithcin on sine wave voltage in a circuit containing R, L, C elements.

Closing switch in time t = 0, to the circuit (*Fig. 12*) sinusoidally variable voltage is switched on

$$u = U_m \sin(\omega t + \varphi_u). \tag{76}$$

After a sufficient period of time from the moment t = 0 in the circuit flows a steady current

$$i_u = \frac{U_m}{Z} \sin(\omega t + \varphi_u - \varphi), \tag{77}$$

where

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}, \quad tg\varphi = \frac{\omega L - \frac{1}{\omega C}}{R} . \tag{78}$$

Solving the systems of differential equations for an aperitic case (62), an critical aperitic (66) and an oscillating (74), we find the transient components of the voltage on the capacitor and current in the inductor (branches of R, L, C).
Equations (76) and (77) determine the components of the temporary voltage on the capacitor and the current in the inductor.
The course of current i and voltage u_C is obtained by adding two courses, respectively i_p, i_u, and u_{Cp}, u_{Cu}.

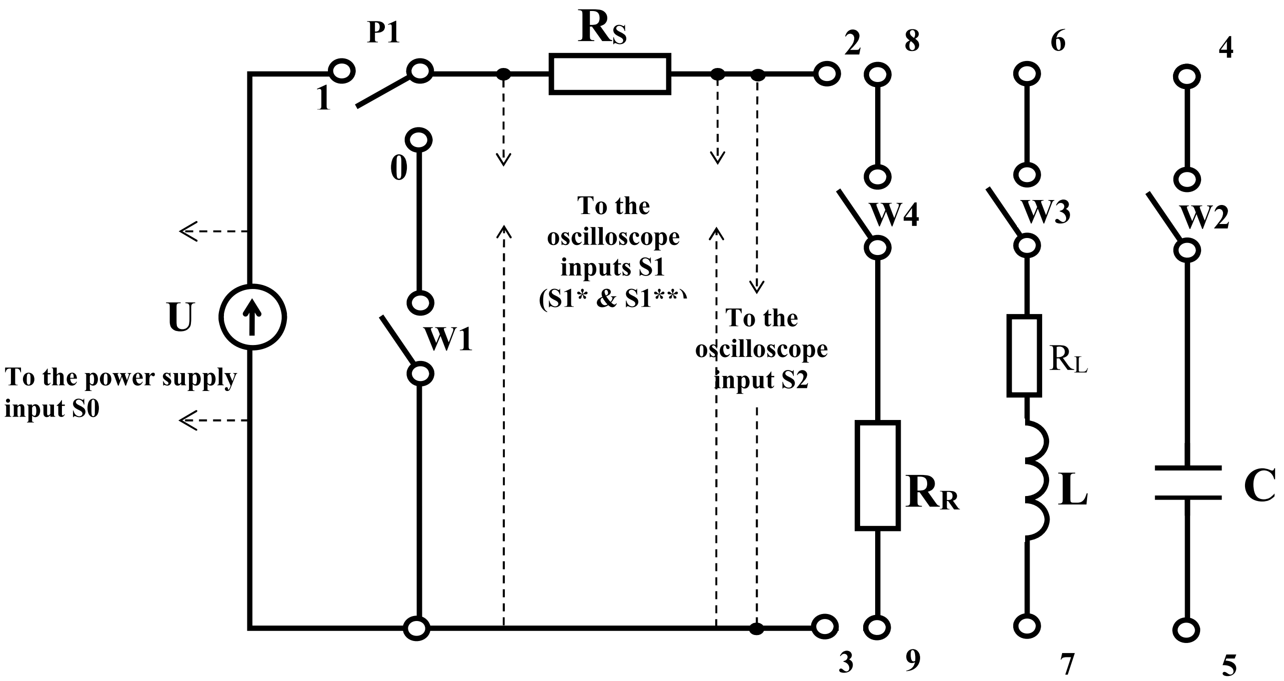
II. Laboratory tests

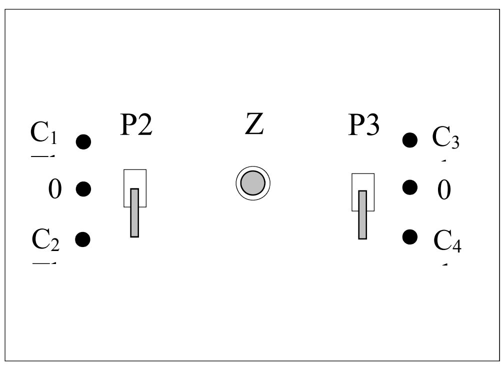
1. Description of the test system

Equipment and components used:

- tested system TRANSIENT STATES
- stabilized power supply (5V)
- set of data acquisition system
  - digital oscilloscope with memory **Hameg**
  - computer with ocilloscope software **SP107**
  - printer

Measuring ciruit





U - *supply voltage (5V)*
R_s - *serial resistor (50Ω)*
L – inductor (*R_l - inductor resistance)*
C - *capacitor*
R_R - *resistor (100Ω)*
W1, W2, W3, W4 - *switches*
P1, P2, P3 - *switches*
Z - *button shorting capacitor*
2, 3, 4, 5, 6, 7, 8, 9 - *connection terminals*
S0, S1, S2 – *BNC sockets, where S0 – power signal, S1 – supply voltage signal (S1* - observed on oscilloscope) and voltage of test element (S1** - observed on oscilloscope), S2 – oscilloscope triggering signal.*

Initial settings:

- P1 at position 0,
- W1, W2, W3, W4 at position OFF (down),
- P2 and P3 at position 0,
- S0 socket connected to the power supply,
- stabilized power supply (5V) enabled,
- switched on computer with the printer,
- switched on the HAMEG oscilloscope,
- output socket S1* (first slot S1) connected to CHI channel,
- output socket S1** (second slot S1) connected to CHII channel,

⇒ <u>for observation of voltage on the test object,</u>

- activate the **CHII** button

(the message is displayed at the bottom of the screen: y2:1V =),

- activate the **SINGLE** button

(preparation of the oscilloscope by pressing the button again – indicated by a light diode **RES** next to the lighted diode **SGL**).

⇒ <u>for observation of current in the circuit (voltage from resistor Rs = 50Ω),</u>

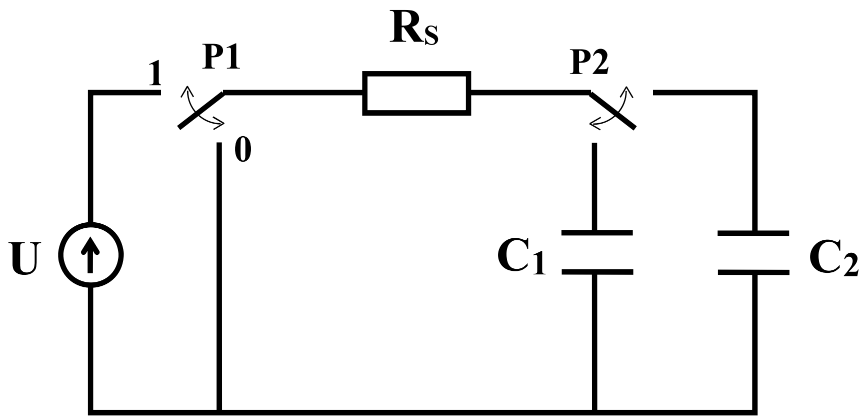
- activate the **CHII** key with the **DU$\overline{\text{AL}}$**button, and the **INV** button.

(on the bottom of the screen should display message: y1:1V= + y2:1V=).
In both channels set the same reinforcements.

- activate the **SINGLE** button (preparation of the oscilloscope by pressing the button again – indicated by a light diode **RES** next to the lighted diode **SGL**),

2. RC circuit tests

During test of the RC circuit, two cases should be used: with capacities C_1 and C_2.
Diagram of the RC circuit test system is shown below.



2.1. Switching on the DC voltage

Steps for observing signals when switching on the DC voltage and to determine the time constant of these signals.

1) Realize the initial settings. Connect terminals 2-4, and 3-5.
   Set the P2 switch to the C_1 position (serial circuit of the R_s and C_1 elements).
   Switch W2 to ON position (up).
2) Set the oscilloscope to observe the appropriate course case.
   Proposed settings for observing the shape of the voltage waveform 1V/cm (CH II), and 20ms/cm, while observing the current waveform 1V/cm (CH I) and 1V/cm (CH II).
3) When the oscilloscope was prepared (the green RES LED is lighted), attach the U = 5V voltage to the system by switching P1 to position 1.
   On the oscilloscope we get the correct course. Switch P1 to position 0.
4) Usinga program running after turning on the computer, read the data from the oscilloscope by selecting the READ option on the monitor screen.
   The transmission is triggered (the RM light on the oscilloscope desktop) which the expected signal will be displayed on the monitor screen.
5) Prepare the received course for printing by selecting the printer icon in the mouse.
   Print a course that is suitable for determining the time constant of the circuit. On the basis of the determined plotting of the constant time and known resistance circuit, determine the capacity of the system.

Perform the above steps for the C_2 capacity (P2 at position C_2).
The proposed setpoints for the time base signal amplification 50ms/cm.

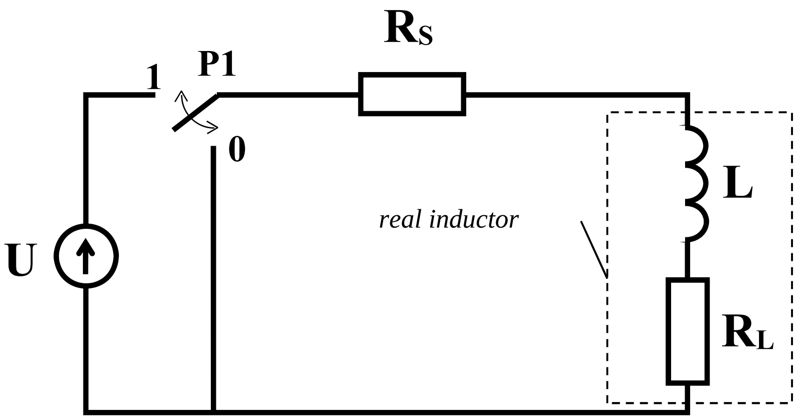
2.2. Discharge of the capacitor

The above steps are also performed for the C_2 capacity (P2 in position C_2).
The proposed setpoints for the time base signal amplification 50ms/cm.

3. RL circuit test

It is practically impossible to perform an ideal inductor, because it will always have a certain internal resistance. The test system used a inductor containing approx. 5000 coils wounded with copper wire with a diameter of 0,25mm^2.
The inductor has a certain R_L resistance and this consequence is impossible to observation voltage only on the L inductance itself.
To minimize the impact of R_L resistance, you can connect additional (parallel) to the inductor resistor R_r= 100Ω. The measurement system is shown below:

3.1. Switching on the DC voltage



Steps for observing signals when switching on the DC voltage, and to determine the time constant of these signals.

1) Realize initial settings. Connect terminals 2-6, and 3-7.
   Switch W3 to ON position (up).
2) Adjust the oscilloscope appropriately to observe the voltage shape between the terminals 2-3, or the current shape of the test circuit. The proposed settings to observe the shape of the voltage waveform 1V/cm (CH II), and 5ms/cm, foro bservations of the current waveform 0.5 V/cm (CH I and CH II).
3) When the oscilloscope was prepared, attach the voltage U = 5V, by switching P1 to position 1. On the oscilloscope screen, we get the right course.
4) Use the computer program to read the data from the oscilloscope by selecting the READ option on the monitor screen. The expected signal will be displayed on the monitor screen.
5) Prepare the received course for printing by selecting the printer icon.

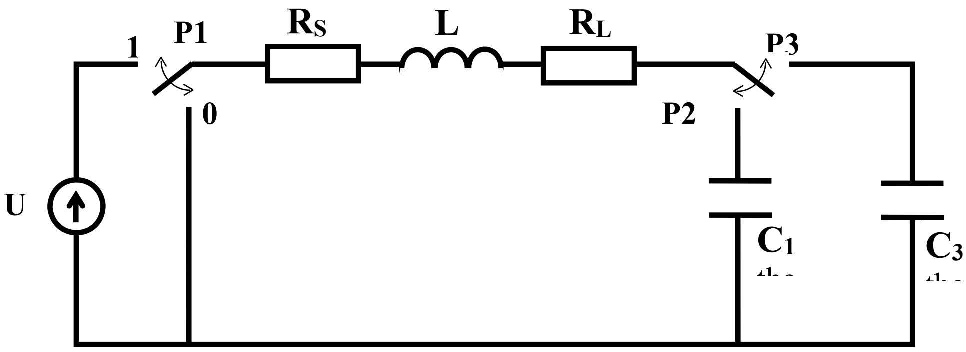
Print a course that is suitable for determining the time constant of the circuit. Based on the determined the time constant and value voltage on the inductor terminals, determine the resistance value R_L, and inductance of the inductor L.

3.2. Short circuit through inductor

The steps for observing the signals during the switch off the DC voltage in the RL system, and for the determination of the time constant of these signals shall be performed analogously as in 3.1. By switching P1 from position 1 to position 0.

4. RLC circuit test

During testing the RLC circuit, two options were used with different capacities for observation of overdamped and underdamped circuits (diagram below).



4.1. Switching on of the DC voltage in the overdamped circuit

Steps for observing signals while switching on the DC voltage.

1) Realize initial settings. Connect terminals 2-6, 7-4 and 5-3.
   Set the P3 switch in the 0 and P2 to position C_1.
   Switch the W2 and W3 switches to ON position (up).
3) Adjust the oscilloscope appropriately to observe the shape of the voltage or current in circuit.
   Proposed settings 1V/cm (CH I, CH II), and 10ms/cm.
   When the oscilloscope is prepared (green RES diode is lighted),
   attach the U = 5V voltage to the circuit by switching P1 to position 1 (up).
   On the oscilloscope we get the correct course. Switch P1 to 0.
4) Using a computer program, read the data from the oscilloscope by selecting the READ option on the monitor screen. The expected signal will be displayed on the monitor screen.
5) Prepare the resulting waveform and print the received waveforms.

4.2. Switching off the DC voltage in the overdamped circuit

The steps to observe the signals must be performed analogously as in section 4.1. by switching P1 from position 1 to position 0.

4.3. Switching on the DC voltage in the underdamped circuit

The steps for observing signals during the switching on the DC voltage in the oscillating circuit perform similarly to the aperitic circuit (4.1), but when the P2 switch is set to pos. 0 and P3 in pos. C_3 (circuit with capacity C_3).

The proposed settings for this case are 1V/cm and 5ms/cm for voltage observation, and 0.5V/cm (CH I), 0.5 V/cm (CH II) and 5ms/cm for current shape observation.

4.4. Switching off the DC voltage in the underdamped circuit

The steps for observing signals when switching off the DC voltage in the oscillating circuit do the same as for the aperitic circuit (4.2), but when setting the P2 switch in pos. 0 and P3 in pos. C_3 (circuit with capacity C_3).

The proposed settings for this case are 1V/cm and 5ms/cm for voltage observation, and 0.5V/cm (CH I), 0.5 V/cm (CH II) and 5ms/cm for current shape observation.

5. Summary of results.

a) for RC and RL circuits.

Circuit type	τ	i_u	u_u	R	L	C
	ms	A	V	Ω	H	μF
RC_1					—	
RC_2					—	
Rl						—

b) for RLC circuits.

Circuit type	s_1	s_2	i_u	u_u	R	L	C
	1/s	1/s	A	V	Ω	H	μF
RLC_1							
RLC_3							

i_u – determined current vaule

u_u – determined voltage value on terminals 2-3,

R, **L**, **C** - total parameters of resistance, inductance and capacitance of the circuit.

⇒ On the basis of measurements and calculations make up both tables.
For inductor circuits, it is important to remember its resistance R_L (i.e. $R = R_S + R_l$).

⇒ For RLC circuit, designate the elements of the characteristic equation (S_1 and S_2), and determine the circuit using equations (57), (58) and (59).

⇒ Perform additional calculations for the specified values of the real circuit parameters: R_S=50Ω, R_L=120Ω, C_1=220μF, C_2=1000μF, C_3=2,2μF, C_4=14,1μF and L=0,4H. Compare the results obtained from the calculations and form the measurements.

III. Conclusions and observations.

The report shall include: computer prints, calculation of characteristic parameters (can be placed under printed courses), tables, conclusions and remarks of the laboratory studies which were carried out.