43. Test of 3-phase systems

The purpose of the laboratory studies is to learn the basic properties of symmetrical and asymmetric star and delta connections in 3-phase systems.

43.1. General information

43.1.1. Types of 3-phase systems

The system of 3 voltages (currents, power) of the same frequency and shifted in the phase by the same angle is called **3-phse system**.
If the amplitudes of these voltages (currents, power) are equal and the phase shift is 2π/3 (120°) the 3-phase system is called **symmetrical.**

The 3-phase voltage is produced in a generator (alternator), having 3 identical windings, called **phase winding**, geometrically shifted by an angle of 120°, rotating with constant speed in the magnetic field. The phases are traditionally indicated by letters: A, B, C; R, S, T or U, V, W. According to the latest Polish standards, the phases labels L_1, L_2, L_3should be used .

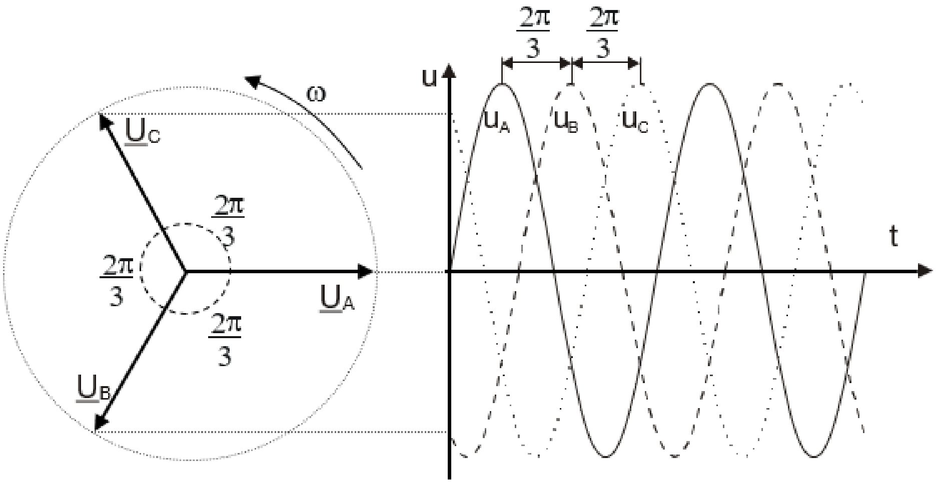

Figure 43. Phasor diagram and waveforms for 3-phase system

Each winding of the generator can be presented in the form of an ideal source of sine wave voltage, therefore the equivalent scheme of the generator presents three voltage sources with the source voltages u_A, u_B, u_C.If we assume that phase B voltage is delayed with relative to phase A by 120° and phase C voltage relative to phase B also by 120°, which is delayed against phase A by 240°, the instantaneous values of the generator voltages are given by formulas:

$$
\begin{aligned}
u_A &= U_m \sin(\omega t + \varphi_u) \\
u_B &= U_m \sin(\omega t + \varphi_u - 120°) \qquad (43.1) \\
u_C &= U_m \sin(\omega t + \varphi_u - 240°)
\end{aligned}
$$

(assuming that the amplitude of the voltage U_m, the pulsations ω, and the voltages phases φ_u are the same in all phase windings).

The complex values[1] of the above voltages are then:

$$\underline{U}_A = U\ e^{j\varphi_u}$$

$$\underline{U}_B = U\ e^{j(\varphi_u - 120^o)} \tag{43.2}$$

$$\underline{U}_C = U\ e^{j(\varphi_u - 240^o)}$$

where: $U = \frac{U_m}{\sqrt{2}}$ - the rms value of these voltages. This 3-phase voltage system with phase sequence: A, B, C **is called a positive sequence.**

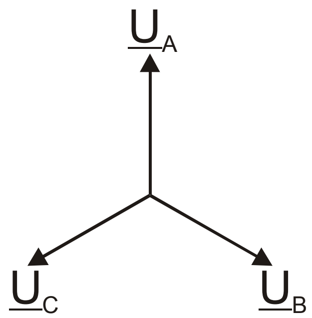

Figure 43. Phasor diagram of the positive sequence of 3-phase voltages system

If phase B voltage leads phase A voltage by 120°, while phase C voltage leads phase B voltage by 120°, so leads of phase A voltage by 240°,then this system is called **a negative sequence.**

[1] *See supplement at the end of chapter.*

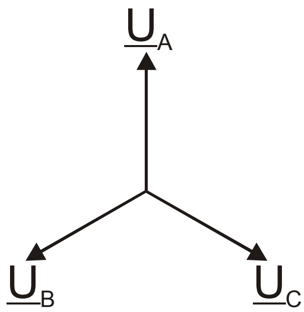

Figure 43. Phasor diagram of the negative sequence of 3-phase voltages system

The 3-phase generator is called **symmetrical** when the voltages on the winding terminals have the same rms values, and the offset between the voltages of the two successive phases is 120°. The symmetrical generator voltages form a positive or negative sequence system.

The wide application of 3-phase systems in the electricity industry results from their advantages, such as:

- reduction of material consumption on the wires when supplying the specified power to the receiver,
- increased by the $\sqrt{3}$ voltage (line voltage), resulting in the receiver being able to operate at a lower current value,
- less power loss in the wires at a given voltage and transmission power (as a result of a reduction of current value),
- production of a rotating magnetic field used in 3-phase motors,

compared to equivalent single-phase systems. Due to the above advantages, the cost of energy transmission in 3-phase systems is considerably lower than for single phase systems.

43.1.2 Determination of the phase sequence for supply grid

Determining the sequence of phases of the voltage supplied to the 3-phase receiver have an important role only when the receiver's work depends on that sequence. A typical example of such receivers are the devices where there is a rotating magnetic field - asynchronous motors. Changing the sequence of phases changes the direction of the motor rotation to the opposite, which may damage the machine.

To determine the sequence of phases are used instruments, called phase sequence indicators. There are 3 basic types of these devices: electromechanical, electronic and equivalent-electric.

The main element of the electromechanical indicator is a miniature 3-phase asynchronous motor, mechanically coupled with a rotating disk. After connecting the testing grid to the respective indicator terminals, the motor starts rotating, together with mechanical connected disk. Right (i.e. clockwise) the direction of rotation of the disk indicates the positive sequence of (according to terminal markings) phases, while the left - of the negative sequence.

The electronic indicator, built on semiconductor elements, is characterized by small dimensions and simplicity operation (compared to electromechanical). It usually has 5 LEDs that indicate the status of the tested grid. The green LED light indicates the positive sequence and the red LED - the negative sequence. The illumination of 3 yellow LEDs indicates the presence of phase voltages.

In the absence of the above indicators, you can build a simple equivalent electrical indicator, which contains two bulbs and a one capacitor (or inductive coil), as shown in Figure 43.5.

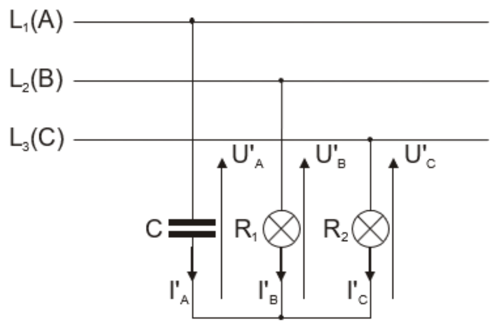

Figure 43. Connection circuit scheme used to determine the sequence of grid phases

Presented condition should be fulfilled for the proper functioning of the circuit:

$$R_1 = R_2 = 1/\omega C$$

or:

$$R_1 = R_2 = \omega L \tag{43.3}$$

where: R_1, R_2 - resistance of light bulbs (in the light state), C - capacitor capacitance, L -inductance of the coil, ω - pulsation of the grid (314 rad/sec).

The capacitor attached to one of the grid phase, which is considered the first (A). After the system has been switched on, due to the capacitance character of phase with capacitor, an asymmetric phase voltage system occurs. The rms voltage value of second phase will be higher than the third phase voltage, which will be signalize by light the bulbs. Thus, brightest light bulb is connected to the second phase (B), and the darkest bulb - to the third phase (C).

If, instead of the capacitor, you apply an inductor connected to the first phase (A), the bulb light is brightest in the third phase (C) and the darkest in the second phase (B).

Instead of light bulbs, you can use resistors that meet the above dependence and measure the voltages on them.

43.1.3 Connection of 3-phase circuits

In practice, symmetrical 3-phase circuits are most common and are powered by symmetrical voltage sources. These circuits are connected (associated) in two basic ways: **star** ⅄ , and **delta △.**

43.1.3.1 Star connection ⅄

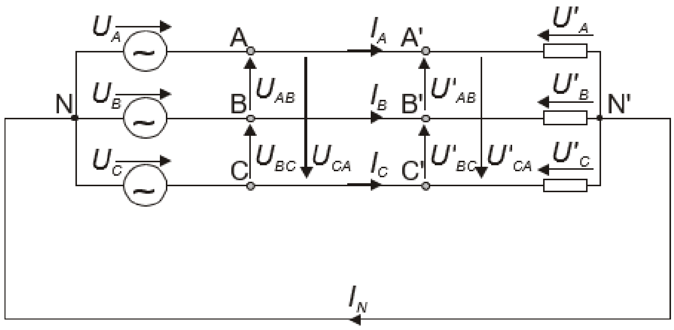

Figure 43.6. Four-wire 3-phase star system

The windings of the 3-phase generator (receiver) are connected to the star when the beginnings of all the windings (output terminals) are connected to each other, and the ends (input terminals) are led outside of the device. The common point of the generator winding is called **the generator's neutral point,** and the common point of the receiver terminals is **the receiver's neutral point**. The wire that connects the generator and receiver's neutral points is called a **neutral wire** (formerly known as zero wire).
The star connection is presented in the Fig. 43.6.
Voltages u'_{A}, u'_{B}, u'_{C} on receiver phases, or voltages u_{A}, u_{B}, u_{C} on the generator phases are called **the phase voltages.** However, the voltages u_{AB} , u_{BC} , u_{CA} between the terminals of the generator, or the voltages u'_{AB} , u'_{BC} , u'_{CA} between the receiver terminals are called **the line voltages**

The currents in the generator or receiver phases are called **the phase currents**, and the currents in the wires connecting the terminals A, B, C of the generator with clamps A ', B ', C ' of the receiver are called line currents.

Based on Figure 43.6, we can write for instantaneous voltages and complex values, respectively

$$u_{AB} = u_A - u_B \qquad u_{BC} = u_B - u_C \qquad u_{CA} = u_C - u_A$$
$$\underline{U}_{AB} = \underline{U}_A - \underline{U}_B \qquad \underline{U}_{BC} = \underline{U}_B - \underline{U}_C \qquad \underline{U}_{CA} = \underline{U}_C - \underline{U}_A \qquad (43.4)$$

and similarly:

$$u'_{AB} = u'_A - u'_B \qquad u'_{BC} = u'_B - u'_C \qquad u'_{CA} = u'_C - u'_A$$
$$\underline{U}'_{AB} = \underline{U}'_A - \underline{U}'_B \qquad \underline{U}'_{BC} = \underline{U}'_B - \underline{U}'_C \qquad \underline{U}'_{CA} = \underline{U}'_C - \underline{U}'_A \qquad (43.5)$$

It is easy to verify that the sum of the complex values of line voltages is always equal to zero:

$$\underline{U}_{AB} + \underline{U}_{BC} + \underline{U}_{CA} = 0 \, . \qquad (43.6)$$

The complex value of the current in the neutral wire is:

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C \, . \qquad (43.7)$$

If in the circuit presented in Fig. 43.6 there is no neutral wire, it is such a system we call the **three-wire system**. In a three-wire system, the sum of the complex values of phase currents is equal to zero:

$$\underline{I}_A + \underline{I}_B + \underline{I}_C = 0 \, . \qquad (43.8)$$

A graphical symbol of a star connection is ⅄ or the letter **Y**.

43.1.3.2 Delta connection △

The windings of the generator (receiver terminals) are connected in a delta when the ends of one winding (output terminal of one phase of the receiver) is connected to the beginning of the next winding (the input terminal of the next phase of the receiver), wherein the output terminals of the generator (input receiver) are the common points of the winding pairs (receiver phases). The delta connection of the generator and receiver shows the Fig. 43.7.

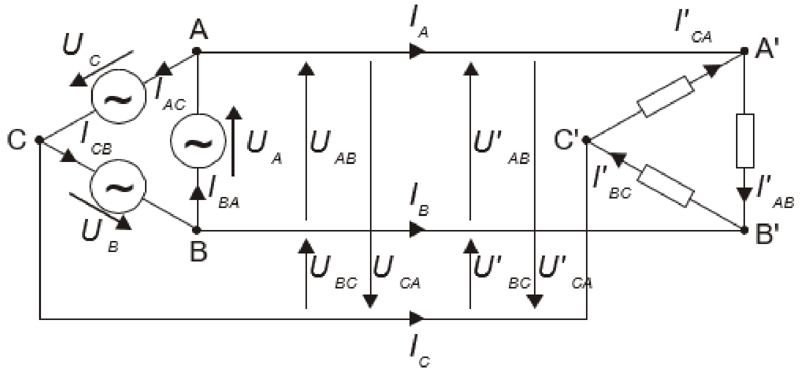

Figure 43.7. 3-phase system with generator and receiver connected in delta

In this system, the values of the line currents, depending on the generator's phase currents or the receiver, and are:

$$\begin{aligned} \underline{I}_A &= \underline{I}_{BA} - \underline{I}_{AC} \\ \underline{I}_B &= \underline{I}_{CB} - \underline{I}_{BA} \\ \underline{I}_C &= \underline{I}_{AC} - \underline{I}_{CB} \end{aligned} \tag{43.9}$$

$$\begin{aligned} \underline{I}_A &= \underline{I}'_{AB} - \underline{I}'_{CA} \\ \underline{I}_B &= \underline{I}'_{BC} - \underline{I}'_{AB} \\ \underline{I}_C &= \underline{I}'_{CA} - \underline{I}'_{BC} \end{aligned} \tag{43.10}$$

The sum of the complex values of the line currents is always zero:

$$\underline{I}_A + \underline{I}_B + \underline{I}_C = 0\ . \tag{43.11}$$

The graphical symbol of the delta connection is **Δ** or the letter **D**.

Currents and voltages in the system outside the generator do not change when the generator with delta connection is replaced by a star connection, provided that the line voltages remain the same.

Therefore, in order to simplify calculations, it is assumed that the generators powering 3-phase circuits are connected to the star.

43.1.4. The relationship between voltages and currents in 3-phase systems

Consider the system connected to the star, as in Fig. 43.6. Assuming a positive sequence of voltages and load symmetry, the phasor diagram of such a system is as follows:

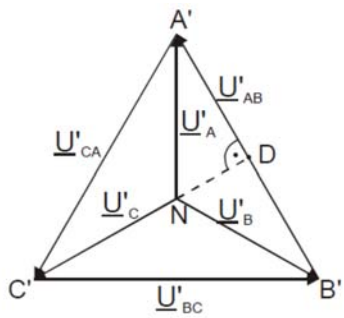

Figure 43.8. The phasor diagram of the symmetrical receiver connected to the star

From the rectangular triangle A′ND we find that:

$$DA' = NA' \cos 30° \quad . \tag{43.12}$$

because:

$$|\underline{U}'_{AB}| = U'_{AB} = 2\,DA', \qquad \cos 30° = \frac{\sqrt{3}}{2}, \tag{43.13}$$

we get:

$$U'_{AB} = \sqrt{3}\,U'_{A} \quad . \tag{43.14}$$

Because in the symmetrical system, the rms phase voltage values are the same, and the rms line voltage values are also the same, we can write in general:

$$U = U\sqrt{3}_{F}$$
$$I = I_f \quad . \tag{43.15}$$

This means that, if $U_f = 230$ V, then $U = 400$ V.

Now consider the symmetrical receiver connected to the delta as presented in Fig. 43.6. Its phasor diagram was presented in Fig. 43.9.

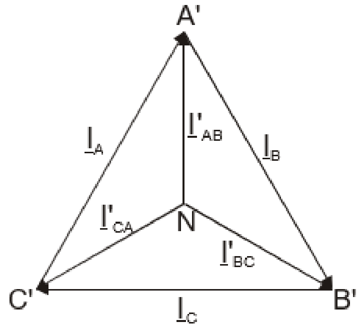

Figure 43.9. The phasor diagram of the symmetrical receiver connected to the delta

On the basis of the above figures we can conclude that the line currents are equal to the difference of the corresponding phase currents:

$$
\begin{aligned}
\underline{I}_A &= \underline{I}'_{AB} - \underline{I}'_{CA} \\
\underline{I}_B &= \underline{I}'_{BC} - \underline{I}'_{AB} \\
\underline{I}_C &= \underline{I}'_{CA} - \underline{I}'_{BC}
\end{aligned}
\tag{43.16}
$$

and generally relationship between phase voltages, currents and line voltages, currents are as follows:

$$
\begin{aligned}
I &= \text{and}\sqrt{3}_{f} \\
U &= U_F
\end{aligned}
\tag{43.17}
$$

43.1.5 Unsymmetrical 3-phase systems

In practice, in addition to discussed above the symmetrical circuits, there are also unsymmetrical systems, both connected to the star and delta. During analyzing such systems, we assume that the power generator is symmetrical and connected to the star, while the unsymmetrical state occurs on the receiver side, due to different impedance of the individual phases, or emergency situations in the supply grid - short circuits or interruptions.

43.1.5.1. Calculation of 3-phase systems.

The method for calculating 3-phase systems is as follows:

- if the receiver is connected to the delta, replace it with an equivalent star,
- calculate the voltage $\underline{U}_N$ between the neutral points N and N ' (star points) of the generator and receiver (see Fig. 43.10),

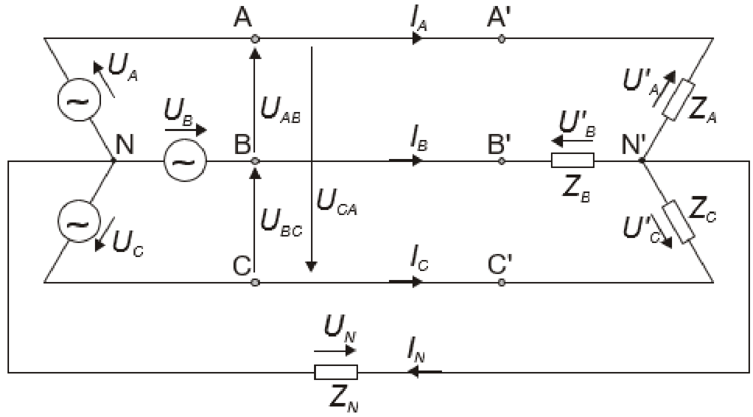

Figure 43.10. Four-wire 3-phase system

Based on the Kirchhoff's laws, it can be demonstrated that:

$$\underline{U}_N = \frac{\underline{Y}_A \underline{U}_A + \underline{Y}_B \underline{U}_B + \underline{Y}_C \underline{U}_C}{\underline{Y}_N + \underline{Y}_A + \underline{Y}_B + \underline{Y}_C} \qquad (43.18)$$

where:

$\underline{Y}_A$, $\underline{Y}_B$, $\underline{Y}_C$ - complex admittances of the receiver phases

$\underline{Y}_N$ – complex admittance of the neutral wire.

In the absence of a neutral wire (three-wire system) $\underline{Y}_N = 0$.
Likewise, in the event of a break in the phase, its admittance is equal to zero.

- based on the calculated voltage $\underline{U}_N$, using the Kirchhoff's laws, we calculate the flow of currents and the distribution of voltages in the analyzed system.

The phase currents of the receiver connected to the star are expressed in the following formulas:

$$\underline{I}_A = \frac{\underline{U}_A - \underline{U}_N}{\underline{Z}_A} \qquad \underline{I}_B = \frac{\underline{U}_B - \underline{U}_N}{\underline{Z}_B} \qquad \underline{I}_C = \frac{\underline{U}_C - \underline{U}_N}{\underline{Z}_C}. \qquad (43.19)$$

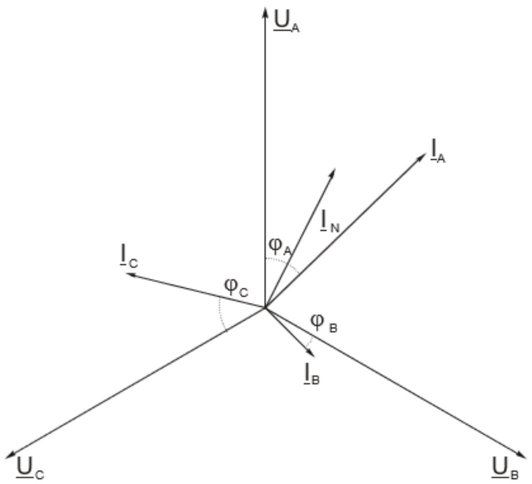

Figure 43.11. Phasor diagram of four-wire unsymmetrical system

43.1.5.2. Break in one of the power wires

A special, unsymmetrical case is the **break** in one of the wires supplying a symmetrical 3-phase receiver. We will consider three cases: a receiver connected in the star, powered by a four-wire system or a three-wire system and a receiver connected in the delta. We assume that the load has a resistive-inductive character– most often found in practice.

a) Four-wire system – receiver connected to the star

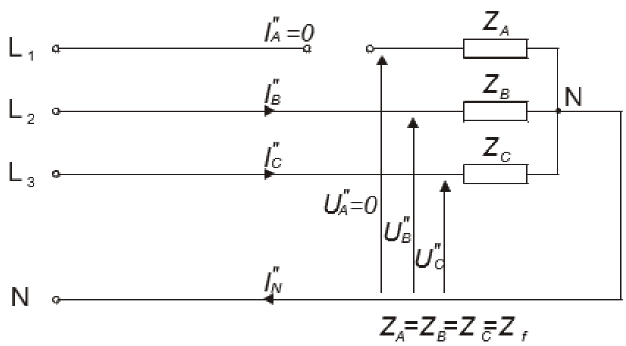

Figure 43.12. Scheme of connection receiver in the star, powered by a four-wire system with break in one wire

Due to interruption in phase A, the current in this phase and the phase voltage are equal to zero. The currents in the remaining phases are:

$$\underline{I}''_B = \frac{\underline{U}''_B}{\underline{Z}_B} = \frac{\underline{U}_B}{\underline{Z}_B} \qquad \text{and} \qquad \underline{I}''_C = \frac{\underline{U}''_C}{\underline{Z}_C} = \frac{\underline{U}_C}{\underline{Z}_C} \tag{43.20}$$

Thus, the currents in the non-damaged phases are the same as under normal operating conditions. Their geometric sum is equal to the complex value of the current in the neutral wire:

$$\underline{I}''_B + \underline{I}''_C = \underline{I}''_N \tag{43.21}$$

The four-wire system scheme is shown in the Fig. 43.13.

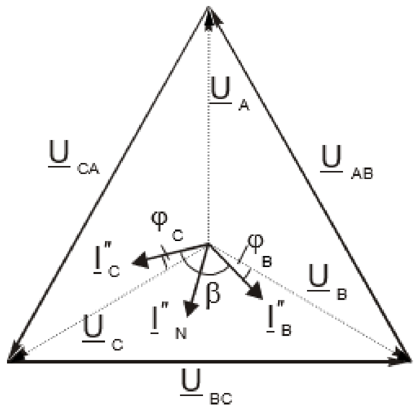

Figure 43.13. Phasor diagram for system in Fig. 43.12.

Based on the diagram, it can be shown that the rms current value in the neutral wire equals:

$$I''_N = \sqrt{I''^2_A + I''^2_C - 2I''_B I''_C \cos(180^o - \beta)} \tag{43.22}$$

where β - the angle between current phasors $\underline{I}''_B$ and $\underline{I}''_C$.

It follows from the above formula that the maximum rms value of the current in the neutral wire occurs at equal loads of both phases, or at the total load of one of them.

b) Three-wire system – receiver connected to the star ⅄

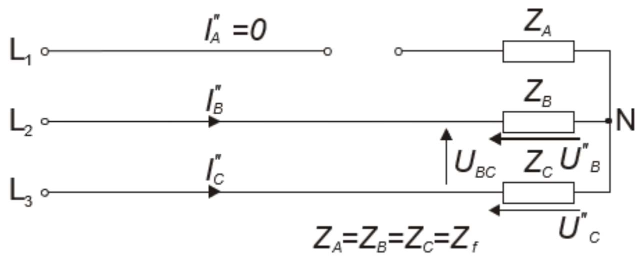

Figure 43.14. Scheme of connection receiver to the star, powered by a three-wire system with break in one wire

It is apparent from the above scheme that the three-wire system with receiver connected in the star after one phase is broken transforms into a two-wire system powered by the line voltage $\underline{U}_{BC}$. Due to the current $\underline{I}''_a = 0$, so the phase voltage $\underline{U}''_a = 0$. The neutral voltage $\underline{U}_N$ of the system is:

$$\underline{U}''_N = \frac{\underline{Y}_A \underline{U}_A + \underline{Y}_B \underline{U}_B + \underline{Y}_C \underline{U}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C} = \frac{\underline{Y}_f (\underline{U}_B + \underline{U}_C)}{2\underline{Y}_f} = \frac{\underline{U}_B + \underline{U}_C}{2}. \qquad (43.23)$$

Hence the phase voltages are equal respectively:

$$\underline{U}_B^{//} = \underline{U}_B - \underline{U}_N^{//} = \underline{U}_B - \frac{\underline{U}_B + \underline{U}_C}{2} = \frac{\underline{U}_{BC}}{2},$$

$$\underline{U}_C^{//} = \underline{U}_C - \underline{U}_N^{//} = \frac{\underline{U}_C - \underline{U}_B}{2} = -\frac{\underline{U}_{BC}}{2}. \qquad (43.24)$$

However, in both other phases it flows the same as the rms value of the phase current, defined by the formula:

$$\underline{I}_B^{//} = -\underline{I}_C^{//} = \frac{\underline{U}_{BC}}{\underline{Z}_B + \underline{Z}_C} = \frac{\underline{U}_{BC}}{2\underline{Z}_f}. \qquad (43.25)$$

The phasor diagram of three-wire system is shown in the Fig. 43.15

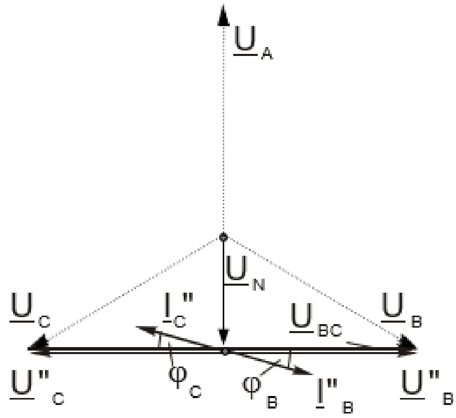

Figure 43.15. Phasor diagram form system in Fig. 43.14.

The rms value of the phase current is therefore:

$$I_f^{//} = \frac{U_{BC}}{2Z_f} = \frac{\sqrt{3}U_f}{2Z_f} = 0{,}87 \times I_f \qquad (43.26)$$

where I_f - the rms value of the phase current in the symmetrical system.

c) receiver connected to the delta △

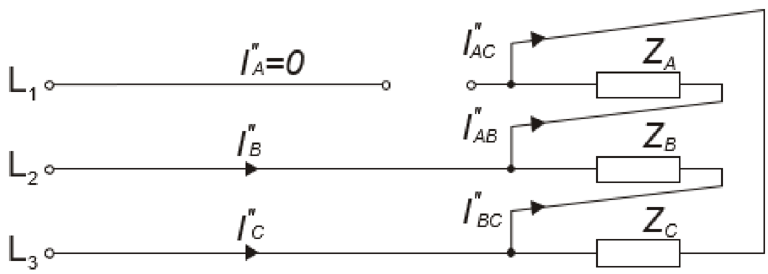

Figure 43.16. Scheme of connection receiver to the delta with interruption in one wire

On the basis of the above scheme we can conclude that the voltage $\underline{U}_{BC}$ and current in the second phase will not change.

$$\underline{I}_{BC}^{//}=\underline{I}_{BC}=\frac{\underline{U}_{BC}}{\underline{Z}_{f}} \quad . \tag{43.27}$$

However, the currents in the first and third phases are the same and equal to half value of the current of the second phase:

$$\underline{I}_{AB}=\underline{I}_{CA}=\frac{\underline{U}_{BC}}{2\underline{Z}_{f}} \quad . \tag{43.28}$$

The phasor diagram of presented system is shown in the Fig. 43.17.

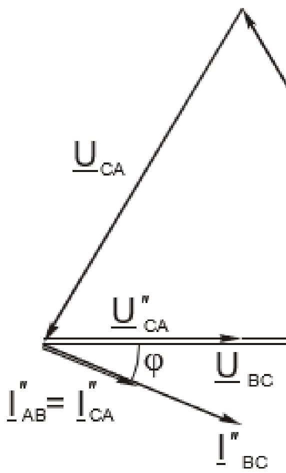

Figure 43.17. Phasor diagram form system in Fig. 43.16.

Due to the broken wire - current phase $\underline{I}_{AB}$, changed by 120°. In a similar way, the current $\underline{I}_{CA}$ phase have changed by 120°. The voltages of first and third phase, as well as currents, have decreased by half value, and their phasors rotated by -120° and + 120° respectively.

$$\underline{U}_{AB}^{//}=\underline{U}_{CA}^{//}=\underline{I}_{AB}^{//}\times\underline{Z}_{f}=\frac{\underline{I}_{AB}}{\underline{Z}_{f}}=\frac{\underline{U}_{BC}}{2} \quad . \tag{43.29}$$

The complex values of the line currents are defined by the formulas:

$$\begin{gathered}\underline{I}_{B}^{//}=\underline{I}_{BC}-(-\underline{I}_{AB}^{//}) , \\ \underline{I}_{C}^{//}=-\underline{I}_{CA}^{//}-\underline{I}_{BC}^{//} \quad .\end{gathered} \tag{43.30}$$

Hence:

$$\underline{I}_B^{//} = \underline{I}_{BC} + \frac{\underline{I}_{BC}}{2} = \frac{3}{2}\underline{I}_{BC} \quad ,$$

$$\underline{I}_C^{//} = -\underline{I}_{BC} - \frac{\underline{I}_{BC}}{2} = -\frac{3}{2}\underline{I}_{BC} \quad . \tag{43.31}$$

43.1.5.3. Short circuit in one of the phases of the receiver connected in a star, powered by a 3-wire system

Another case of unsymmetrical system is a **short-circuit** in one of the phases of the receiver connected to the star, powered by a 3-wire system. The scheme of this case shows the Fig. 43.18.

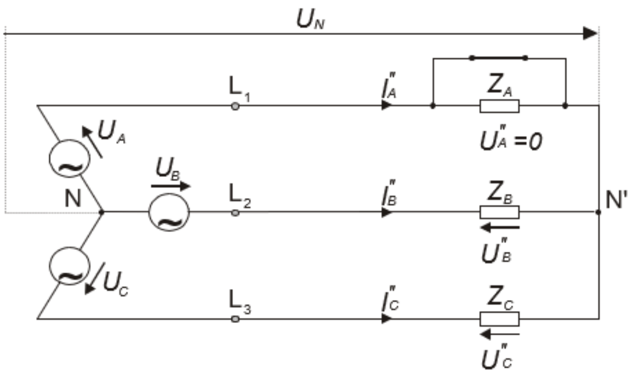

Figure 43.18. Short-circuit in phase A of receiver connected to the star in a 3-wire system

Based on the formulas (43.19) the phase currents values prior to the occurrence of a short circuit were as appropriate:

$$\underline{I}_A = \frac{\underline{U}_A - \underline{U}_N}{\underline{Z}_A} \qquad \underline{I}_B = \frac{\underline{U}_B - \underline{U}_N}{\underline{Z}_B} \qquad \underline{I}_C = \frac{\underline{U}_C - \underline{U}_N}{\underline{Z}_C} . \tag{43.32}$$

After short circuit in phase A, the voltage between the generator and receiver neutral points becomes equal to the voltage $\underline{U}_A$:

$$\underline{U}_N = \underline{U}_A \quad . \tag{43.33}$$

Substituting this dependence to formulas (43.32) we receive:

$$\underline{I}_B^{//} = \frac{\underline{U}_B - \underline{U}_A}{\underline{Z}_B} = -\frac{\underline{U}_A - \underline{U}_B}{\underline{Z}_B} = -\frac{\underline{U}_{AB}}{\underline{Z}_B}$$
$$\underline{I}_C^{//} = \frac{\underline{U}_C - \underline{U}_A}{\underline{Z}_C} = -\frac{\underline{U}_A - \underline{U}_C}{\underline{Z}_C} = -\frac{\underline{U}_{AC}}{\underline{Z}_C} \quad . \tag{43.34}$$

from the I Kirchhoff's law we receive:

$$\underline{I}_A^{//} = -(\underline{I}_B^{//} + \underline{I}_C^{//}) = -(-\frac{\underline{U}_{AB}}{\underline{Z}_B} - \frac{\underline{U}_{AC}}{\underline{Z}_C}) = \frac{\underline{U}_{AB}}{\underline{Z}_B} + \frac{\underline{U}_{AC}}{\underline{Z}_C} \; . \tag{43.35}$$

It is apparent from the above equations that the phase B and C receivers will have an line voltage (400 V in the case of a phase voltage 230 V) which may cause damage. The phasor diagram is presented in the Fig. 43.19.

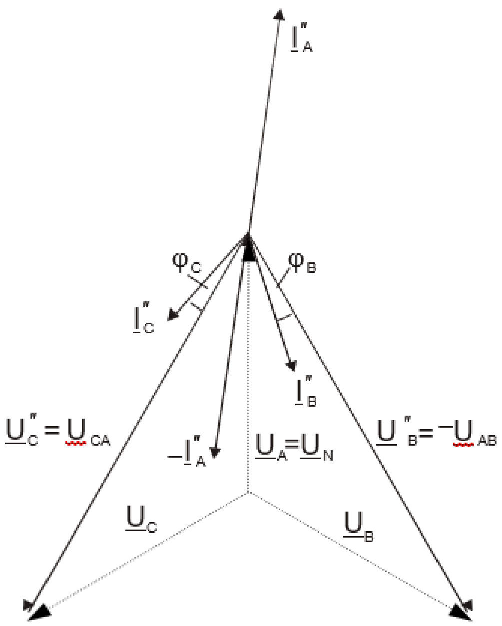

Figure 43.19. Phasor diagram of receiver connected in the star with a short circuit in one phase in 3-wire system.

43. Laboratory tests.

Diagram of the measuring system for tests of the 3-phase system is presented in Fig. 43.20.

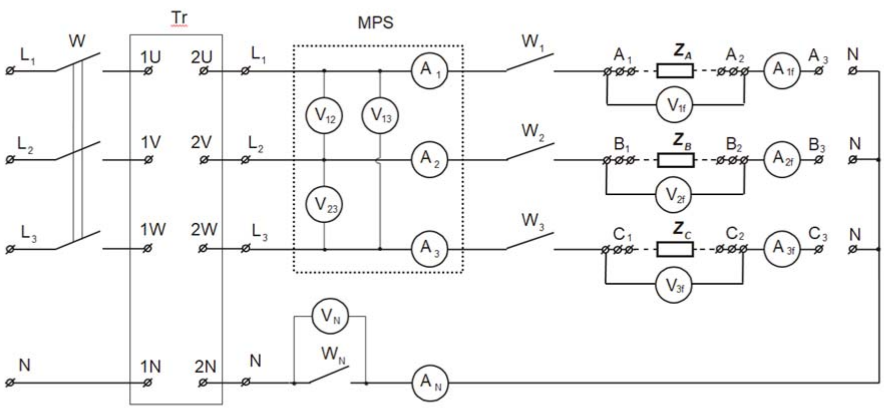

Figure 43.20. Diagram of the measuring system for tests of the 3-phase system

Marks:

V_{1f}, V_{2f}, V_{3f}, V_N – digital voltmeter

A_{1f}, A_{2f}, A_{3f}, A_N – digital ammeters

W, W_1, W_2, $W_{3,N}$ – circuit breakers

Z_A, Z_B, Z_C – phase impedances of 3-phase receiver

Tr – 3-phase transformer to decreased the voltage value

MPS – Measuring Parameters System
(line voltages measurement: V_{12}, V_{13}, V_{23}
line currents measurement: A_1, A_2, A_3)

Note: The values of the phase currents **should not exceed 1 A** due to technical limits resulted of used slide resistors.

43.2.1 Determination of the phase sequence for supply grid

To determine the phase sequence of L_1, L_2, L_3 connect the equivalent phase sequence indicator (the capacitor connect to phase L_1).

43.2.2 Test of the receiver connected in the star.⅄

Between pair of terminal $A_1 - A_2$, $B_1 - B_2$, $C_1 - C_2$ connect phases of receiver Z_A, Z_B, Z_C according to the Fig. 43.21.

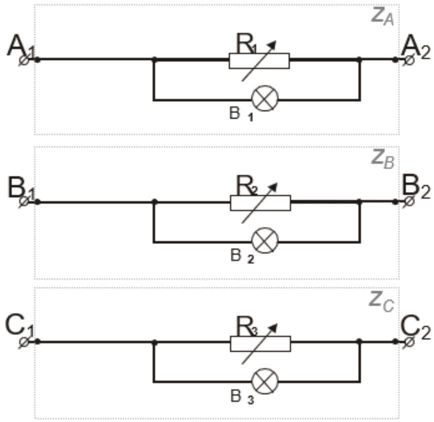

Figure 43.21. Receivers in 3-phase system

<u>Labels:</u> R_1, R_2, R_3 – slide resistors,
B_1, B_2, B_3 – signaling bulbs.

Slide resistors are used as the receiver in the laboratory tests, coupled in parallel with the lighting bulbs. Resistors allow for smooth adjustment of the rms value of the phase currents. Such receivers Z_A, Z_B, Z_C have resistance character.

The receivers have to be connected in the star in the systems (Fig. 43.21), short points A_3, B_3 and C_3 with each other, or by connecting these points with the point N of the receiver (4- wire system).

Measurements shall be performed for all currents, voltages and power in the following cases:

1) **4-wire** **<u>symmetrical</u>** system (Switch on: W_1,W_2,W_3,W_N and W)
Set resistors to obtain same values of phase currents.

2) **4-wire** system **<u>with a break in one phase</u>**.
Open the switch in the selected phase wire
e.g. W_1. (Switch on: W_2, W_3, W_N and W)

3) **3-wire** **<u>symmetrical</u>** system. Switch on W_1.
Open the switch W_N. (Switch on: W_1,W_2,W_3 and W)

4) **3-wire** system **<u>with a break in one phase</u>**.
Open the switch in the selected phase line
e.g. W_1. (Switch on: W_2,W_3 and W)

5) **3-wire** system **<u>with short circuit in one phase</u>**.
Open the switches: W, W_1,W_2,W_3. Using wire short selected receiver.
e.g. A_1 with A_2. Make sure that the switch W_N is not closed. Power on. (Switch on: W_1,W_2,W_3 andi W)).

The results of the measurements note in table 1a and 1b.
Line values of currents and voltages read from the Measuring Parameters System (MPS), and voltmeter V_N and ammeter A_N.
The phase values of the current and voltage of the receiver read from voltmeters V_{1f} , V_{2f} , V_{3f} and ammeters A_{1f} , A_{2f} , A_{3f} .
The power values retrieved by the receivers are read from Measuring Parameters System (MPS)

Table 1a.
Line parameters

	U_{12} [V]	U_{23} [V]	U_{31} [V]	U_N [V]	I_1 [A]	I_2 [A]	I_3 [A]	I_N [A]
1.								
2.								
3.								
4.								
5.								

Table 1b.
Phase parameters

	U_{1P} [V]	U_{2P} [V]	U_{3P} [V]	I_{1P} [A]	I_{2P} [A]	I_{3F} [A]	P_{1P} [W]	P_{2P} [W]	P_{3P} [W]
1.									
2.									
3.									
4.									
5.									

Based on obtained results:

a) desrcibe the relationship between the phase and line voltages of the symmetrical system,
b) calculate the impedance of load of each phase,
   using the following formulas:

$$Z_P = \frac{U_P}{I_P}, \qquad \varphi_P = \arccos \frac{P_P}{U_P \times I_P}, \tag{43.36}$$

where:

P_P - measured power values, or values calculated from the formula: $P_P = U_P \times I_P$ for load with resistance character (cosφ= 1)*,*
U_P -indication of the voltmeter measuring voltage at the load of the selected phase,
I_P -indication of the ammeter measuring current of the selected load phase,
Z_P -module of the total load impedance of the selected phase,
φ_P -the angle between the voltage and current indication of the selected phase.

c) draw phasors diagrams of voltages and currents for the measuring points selected by the teacher.

Results of the calculation note in table 2 and provide example calculations.

Table 2.

A summary of the calculation results for the receiver connected in the star.

	Z_A	φ_A	Z_B	φ_B	Z_C	φ_C
	[Ω]	[°]	[Ω]	[°]	[Ω]	[°]
1.						
2.						
3.						
4.						
5.						

43.2.3 Test of the receiver connected in the delta △.

The receivers have to be connected in the delta in the systems (Fig. 43.21) by short the corresponding points. Since there may be two possibilities to connect to the delta - you need to note which points were connected (e.g. ammeter A_{1F} can measure current I_{12} or I_{13}). For proper operation of the MPS meter, it is necessary to switch of W_N.

Measurements shall be performed for all currents, voltages and power in the following cases:

1) **3-wire <u>symmetrical</u>** system.
Set the same values for the phase currents. (Switch on: W_1,W_2,W_3 and W)

2) **3-wire** system **<u>with break in one phase</u>**. Open the switch in the selected phase line e.g. W_1. (Switch on: W_2,W_3 and W)

The results of the measurements note in table 3a and 3b:

Table 3a.

Line parameters

	U_{12}	U_{23}	U_{31}	I_1	I_2	I_3
	[V]	[V]	[V]	[A]	[A]	[A]
1.						
2.						

Table 3b.

Phase parameters

	U_{1P}	U_{2P}	U_{3P}	I_{1P}	I_{2P}	I_{3F}	P_{1P}	P_{2P}	P_{3P}
	[V]	[V]	[V]	[A]	[A]	[A]	[W]	[W]	[W]
1.									
2.							-	-	-

Based on obtained results:

a) verify the theoretical relationship between the rms phase and line voltages, currents values of the receiver connected to the delta,

b) calculated on the basis of the indications of voltmeters and ammeter the total load impedance of each phase (according to 43.36),

c) draw phasors diagrams of voltages and currents for the measuring points selected by the teacher.

Results of the calculation note in table 4, and provide example calculations:

Table 4

Summary of calculation results for the receiver connected to the delta.

	Z_A	φ_A	Z_B	φ_B	Z_C	φ_C
	[Ω]	[°]	[Ω]	[°]	[Ω]	[°]
1.						
2.						

43. Remarks and conclusions.

The report shall include: tables with results of the measurement and calculation, example of calculations, selected phasor diagrams, comments on the course of the exercise and discussion on the results obtained. Compare the results with theoretical dependencies.

Literature

[1] Krakowski M.: Theoretical electrical Engineering. Volume 1: Linear and nonlinear circuits. PWN, Warsaw, 1999.

[2] Collective work "Electrical Engineering and electronics for non-electricians". WNT, Warsaw, 1995.

Representation of sinusoidal quantities using complex numbers

The calculation of sinusoidal current circuits is considerably simplified by using complex numbers. We will signify the unity of the imaginary by *J* ; used in mathematics the designation of the unity of the imaginary symbol *i* is inconvenient in electrical engineering, because it means a temporary value of current.
The complex number is presented in the form:

$$\underline{z}=a+jb,$$

where $a = \mathrm{Re}\{\underline{z}\}$ is the real part, and $b = \mathrm{Im}\,\underline{z}\}$ is the imaginary part of the complex number.
The above expression of a complex number is an algebraic form. The complex number can also be presented in an exponential form:

$$\underline{z} = z\, e^{j\alpha},$$

or trigonometric:

$$\underline{z} = z\,(\cos\alpha + j\,\sin\alpha),$$

wherein $z = \sqrt{a^2 + b^2}$ is a complex number module, and $\alpha = \tan^{-1}\frac{b}{a}$ the complex number argument.
The geometric image of the complex number z is on the plane of the complex variable vector **OA** (see figure).

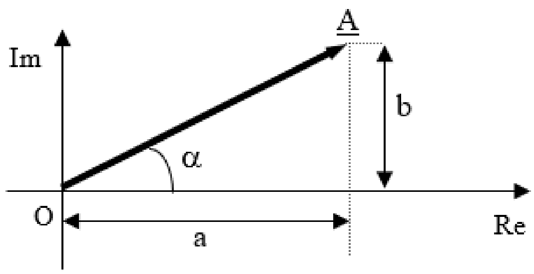

Figure 43.22. Geometric image of a complex number.

The transition from exponential to trigonometric form allows Euler's formula:

$$e^{j\alpha}=\cos\alpha+j\,\sin\alpha\,.$$

Based on the above formula, taking into account that 2π is the period of trigonometric function, we receive

$$e^{j(\alpha+k\cdot 2\pi)} = e^{j\alpha}, \qquad k=1, 2, ...,$$

The result is that the complex number argument is not explicitly specified, but it takes a value that differs by any multiple of angle 2π.
Based on the above analysis, the voltage $u=U_m \sin(\omega t+\varphi_u)$, and the current $i=I_m \sin(\omega t+\varphi_i)$, can be represented in a complex form as follows:

$$\underline{U}_{mt} = U_m e^{j(\omega t+\varphi_u)}, \text{ and } \underline{I}_{mt} = I_m e^{j(\omega t+\varphi_i)}.$$

The instantaneous values of u and I are obtained by extracting the imaginary part of the expression. $u=\mathrm{Im}\{\underline{U}_{mt}\}$, $i=\mathrm{Im}\{\underline{I}_{mt}\}$.
The complex (symbolic) values of voltage and current determine the corresponding expression:

$$\underline{U} = U e^{j\varphi_u}, \text{ and } \underline{I} = I e^{j\varphi_i}.$$

The module of the complex value and its argument equal the rms value and the phase sinusoidal waveform respectively.